Income distribution orderings based on differences with respect to the minimum acceptable income

by

ALAITZ ARTABE ECHEVARRIA[1]

Master´s thesis director

JOSÉ MARÍA USATEGUI[2]

Abstract

This paper analysis the possibilities of using the indexes of riskiness of Aumann-Serrano and Foster-Hart to order income distributions, taking into account the differences with respect to the minimum acceptable income level.

[1] Financial support from the "Beca para formación de personal investigador" of the Department of Education, Universities and Research of the Goverment of the Basque Country.

[2] Departamento de Fundamentos del Análisis Económico II, Universidad del País Vasco/Euskal Herriko Unibertsitatea

1.-INTRODUCTION

The distribution of income has been studied in many papers because economists want to know which distribution is more adequate. In the literature, different types of indexes have been used in order to evaluate income distributions. Some of them, as the index of Atkinson, incorporate the preferences of the individuals and they take into account a social welfare function. Other indexes, as the inequality indexes (Gini,….), are not built taking into account any social welfare function, but incorporate implicitly some social preferences as they give different weights to individuals.

The aim of this paper is to analyze the possibilities of using the indexes of riskiness of Aumman-Serrano (2008) and Foster-Hart (2008) to compare income distributions taking into account the levels and the dispersion of the incomes. Those indexes are used to measure the riskiness of risky assets taking into account the possible values that the asset can take and using the initial income (or wealth) level as the point of departure.

An analogous context in the case of income distributions requires the definition of the level of income that will be used as the point of comparison. The distributions of income are then evaluated from the point of view of an egalitarian society where each individual has an income level equal to that income level used as the point of comparison. Alternatively, we can consider that the distributions of income are evaluated from the point of view of an individual with an income level equal to that income level used as the point of comparison. In both approaches, the individual does not know the difference with respect to the income level used as the point of comparison that he will obtain, and this difference can be positive or negative. The individual only knows the probability of obtaining each level of income (he is behind the veil of ignorance with respect to his income level). The indexes proposed in this way will be a complete order in the set of income distributions, so they will allow us to order and compare all the income distributions.

It will be considered in this work that to get an adequate life level it is necessary to have a minimum acceptable income level and that this minimum acceptable income level is greater than the income level required for survival. The minimum acceptable income level will be the income level used as the point of comparison. The differences or dispersions of the incomes in the distribution will be calculated with respect to this minimum acceptable income level and these differences will be used to define the indexes. Individuals can have gains or losses with respect to the minimum acceptable income level (incomes may be higher or lower than the minimum acceptable income level), but we consider that the expectation of the income distributions considered are greater than the minimum acceptable income level.

The paper is organized in the following way. Section 2 contains the presentation of the indexes of Aumann-Serrano and Foster-Hart as riskiness indexes and their properties. Section 3 discusses the possible extension of these indexes to the case of income distributions. The last part of that section compares the extensions of those indexes to the case of income distributions with the index of Gini. Section 4 includes some results that would have to be taken into account if the proposed extensions of the indexes of riskiness were used to compare income distributions. The last section of the paper is an illustration of the extensions of the indexes of Foster-Hart and Aumann-Serrano to order income distributions using the data corresponding to the income distributions of the years 1995 and 2002 in Spain.

2.- PRESENTATION OF THE INDEXES OF AUMANN-SERRANO AND FOSTER-HART.

The indexes of Aumman-Serrano and Foster-Hart were built to measure the risk of risky assets and investments, taking into account the dispersions that the possible results of these investments have with respect to the initial point or situation.

Consider that the individual has an initial wealth, W, and he has to decide between accepting or not a risky asset. A risky asset or gamble g is a random variable with real values. Some of these values are negative, $P[g<0]>0$, but the gamble g has positive expectation, $E(g)>0$. This gamble indicates the possible values that the risky asset can take if the individual accepts it and each value is associated with one probability. The probabilities assigned to each value in the gamble g are given by the variable p.

2.1.- Index of Aumann-Serrano

The index of riskiness of Aumann-Serrano is presented in the paper "An Economic Index of Riskiness" (forthcoming in the Journal of Political Economy). They characterize the riskiness of a gamble in a way that implies that it is equal to the reciprocal of the coefficient of absolute risk aversion of the individual with constant absolute risk aversion (CARA) who is indifferent between taking and not taking that gamble. They note that CARA individuals with lower coefficient of absolute risk aversion are willing to take riskier gambles.

Aumann and Serrano sort risky assets taking into account the gains and losses that they can provide, g, and also the probability of obtaining each value, p. They use, in the context of the Expected Utility Theory of von Neumman-Morgensten, a Bernouilli utility function for money, which is strictly monotonic, strictly concave, twice continuously differentiable and defined over the real line. It is considered that an agent with utility function u accepts a gamble g at wealth W if E[u(W+g)]>u(W); otherwise he rejects gamble g.

An individual that is very risk-averse would not accept a risky asset but someone with low risk-aversion would accept it, because the decision of accepting or not depends on the degree of risk aversion. Aumann and Serrano determine the risk aversion coefficient of the CARA individual who is indifferent between accepting or not the risky asset.

The index built by Aumann and Serrano is:

$$E\left[e^{\frac{-g}{R(g)}}\right]=1$$

They call R(g) the riskiness of g.

If a constant absolute risk averse (CARA) agent accepts a gamble, then any CARA agent with a smaller coefficient of absolute risk aversion would also accept that gamble. Equivalently, if a CARA agent rejects a gamble, then any CARA agent with a larger coefficient of absolute risk aversion would also reject that gamble. So, for each gamble, there is a value of the coefficient of absolute risk aversion such that g is accepted by CARA agents with smaller coefficient of absolute risk aversion, but rejected by CARA agents with larger coefficient of absolute risk aversion.

In this case, the index of riskiness, R(g), of the risky asset g is the reciprocal of the number α such that a CARA person with coefficient of absolute risk aversion α is indifferent between taking and not taking the risk.

2.2.- Index of Foster-Hart

The index of Foster-Hart is presented in the paper "An Operational Measure of Riskiness". They propose a measure of riskiness of "gambles" that is based on the *critical wealth* level below which it becomes "risky" to accept the gamble. They sort the risky assets building a measure of riskiness of each asset that considers the circumstances under which the individual would risk bankruptcy if he accepts the asset. Foster and Hart consider that the measure has to depend on the gamble itself but not on the decision-marker, that is, they consider that the outcomes and the probabilities of the gambles are the only aspects that matter. They note that the risk of accepting g clearly depends on the wealth of the individual.

The riskiness of a gamble g is defined as the *critical wealth* below which accepting g becomes risky. This *critical wealth level* is the threshold that distinguishes between situations where it is "risky" to accept g and those where it is not risky to accept g, and that critical wealth level depends only on the distribution of the gamble. *There is a critical wealth function Q that associates to each gamble g a number Q(g) such that a gamble g is rejected at wealth W if W<Q(g), and is accepted if W≥Q(g)*. So, Q(g) is the minimal wealth at which g is accepted.

This index has to do with the idea of *no-bancruptcy*. If we assume that the initial wealth is positive and borrowing is not allowed, *bancruptcy* occurs when the wealth becomes zero, or more generally, when it converges to zero. And the strategy s yields no-bancruptcy for the process (sequence of gambles) G and the initial wealth W_1 if the probability of bancruptcy is zero, $P\left[\lim_{t\to\infty} W_t = 0\right] = 0$.

Foster and Hart proved that *for every gamble g there exists a unique real number R(g)>0 such that a simple strategy with critical-wealth function Q guarantees no-bancruptcy if and only if Q(g)≥R(g) for every gamble g. Moreover, R(g) is uniquely determined by the equation:*

$$E\left[\log\left(1+\frac{1}{R(g)}g\right)\right]=0$$

So, a simple strategy s guarantees no-bancrupty if and only if for every $g \in G$, s rejects g at all W<R(g). R(g) is the initial wealth at which the individual is indifferent between accepting or not the risky asset. If the initial wealth W is higher than R(g) the individual will accept the asset, and if W<R(g) the individual will reject the asset.

If the Bernoulli utility function is logarithmic, the analysis of Foster-Hart is equivalent to rejecting any game that any individual with logarithmic utility function (that is, with constant relative risk aversion, CRRA, and coefficient of relative risk aversion equal to 1) would reject.

2.3.- Properties of the indexes

These indexes satisfy several properties. In both indexes, the riskiness of a gamble g only depends on the distribution of the gamble.

2.3.1-AUMANN-SERRANO

i) MONOTONIC WITH RESPECT TO STOCHASTIC DOMINANCE: g first-order stochastically dominates h if $g \geq h$ for sure, and $g > h$ with positive probability, and g second-order dominates h if h may be obtained from g by "mean-preserving spreads". The riskiness index of Aumann-Serrano is monotonic in both senses.

ii) CONTINUITY: The riskiness index R is continuous, because when two gambles are likely to be close, their riskiness levels are close.

iii) DILUTED GAMBLES: If g is a gamble, p a number strictly between 0 and 1, and g^p a compound gamble that yields g with probability p and 0 with probability 1-p, then $R(g^p)=R(g)$.

iv) COMPOUND GAMBLES: If two gambles g and h have the same riskiness r, then a compound gamble yielding g with probability p and h with probability 1-p also has riskiness r.

v) NORMAL GAMBLES: If the gamble g has a normal distribution, the
$$R(g) = \frac{Var(g)}{2E(g)}$$

vi) SUMS OF GAMBLES: If g and h are independent identically distributed gambles with riskiness r, the g+h also has riskiness r. So, the sum of n i.i.d. gambles has the same riskiness as each one separately.
If g and h are not independent, we still have subadditivity, so, $R(g+h) \leq R(g) + R(h)$, for every gamble g and h.

vii) THE DOMAIN: When $E(g)>0$, the range of the index of riskiness will be between 0 and ∞.

viii) LOSSES: The riskiness index R is more sensitive to losses than to gains.

2.3.2.-FOSTER-HART

For all gambles g,h $\in G$, the following properties are satisfied (see Foster and Hart (2008), section 5):

i) DISTRIBUTION: If g and h have the same distribution then $R(g)=R(h)$.

ii) HOMOGENEITY: $R(\lambda g)=\lambda R(g)$ for every $\lambda>0$

iii) MAXIMAL LOSS: $R(g)>L(g)$

iv) SUBADDITIVITY: $R(g+h)\leq R(g)+R(h)$

v) CONVEXITY: $R(\lambda g+(1-\lambda))\leq \lambda R(g)+(1-\lambda)R(h)$ for every $0<\lambda<1$.

vi) DILUTION: $R(\lambda * g)=R(g)$ for every $0<\lambda<1$

vii) INDEPENDENT GAMBLES: If g and h are independent random variables then $\min\{R(g),R(h)\}<R(g+h)<R(g)+R(h)$

viii) MONOTONIC WITH RESPECT TO STOCHASTIC DOMINANCE: If g first order stochastically dominates h, or if g second-order stochastically dominates h, then $R(g)<R(h)$. The index is monotonic with respect to stochastic dominance because a gamble that dominates another has a lower riskiness. Our indexes are of *total-order*, because we can use them to compare all pairs of distributions, but the stochastic dominance is a *partial-order* because we can only compare some pairs of distributions.

3.- EXTENSIONS OF THE RISKINESS INDEXES TO THE CASE OF INCOME DISTRIBUTIONS

The aim of this paper is to analyze the possibilities to adapt the indexes of Aumann-Serrano and Foster-Hart to *order* income distributions. We will use these indexes to compare income distributions taking as the initial point the "minimum acceptable income level".

The main difference of this application with respect to the case of risky assets is that in the case of risky assets the individual has to decide to accept or not to accept a risky asset and if he rejects the asset, he will have the initial wealth level. But in the case of income distributions all the possible income levels will happen. It is considered that the distributions of income are then evaluated from the point of view of an egalitarian society where each individual has an income level equal to the minimum acceptable income level. Alternatively, we can consider that the distributions of income are evaluated from the point of view of an individual with an income level equal to the minimum acceptable income level. In both approaches, the individual does not know the difference with respect to the income level used as the point of comparison that he will obtain, and this difference can be positive or negative. The individual only knows the probability of obtaining each level of income (he is behind the veil of ignorance with respect to his income level).

Let us denote the "minimum acceptable level of income" by $\hat{Y}$, It may not be easy to know which level of income is the minimum acceptable in a particular society. The indexes we are going to present are defined using this "minimum acceptable level" as the point of reference. There is also a "minimum level of income needed to survive", Y^s, that is lower than the minimum acceptable income level ($Y^s < \hat{Y}$).

The distribution of income is Y and the level of income of an individual is Y_i. The difference between the income level of the individual i and the "minimum acceptable income level" is g_i.

We will consider that $E(Y_i) > \hat{Y}$ or equivalently, $E(g)>0$, so the average level of income of the society is higher than the "minimum acceptable income level". This condition is necessary to have a solution for each index.

It is also considered that there is at least one person whose income level is lower than the "minimum acceptable income level", $\min(g)<0$.

3.1.-Extension of the index of Aumann-Serrano

We will apply the index of A-S to sort income distributions taking into account the income dispersion with respect to the minimum acceptable income level. The application of the index of Aumann-Serrano is:

$$E\left[e^{\frac{-g}{R(g)}}\right]=1$$

where g is the gamble given by the differences between the possible values that the income levels can take and the minimum acceptable income level.

In order to sort different income distributions taking into account this index, we need to calculate the value of R(g) for each income distribution. The higher the value of the index is, the worse is the income distribution (to accept the change to the income distribution considered, a lower constant absolute risk aversion coefficient is required in a CARA egalitarian society with income for each individual equal to the minimum acceptable income level and individuals behind the veil of ignorance).

So if there are two income distributions, g and h, and R(g)>R(h), the income distribution h is better than the income distribution g.

3.2.- Extension of the index of Foster-Hart

The index of Foster-Hart is:

$$E\left[\log\left(1+\frac{1}{R(g)}g\right)\right]=0$$

and it has to do with the idea of *bankruptcy*. In the case of risky assets, the individual will use an acceptation rule such that the application of that rule to a sequence of games avoids bankruptcy. In this sequence of games the new income obtained after a game is the point of departure to evaluate the next game. In the case of income distributions, however, all incomes in the distribution do occur.

We may use the extension of the index of Foster-Hart to order different income distributions. So the higher is the value of R(g), the income distribution will be worse. Given two income distributions, g and h, if R(g)>R(h) the distribution of h would be better than the distribution of g.

The extension of the index of Foster-Hart may be used also to consider acceptable only income distributions that guarantee (in a sequence of new income distributions obtained from any of the incomes that may be obtained in the initial income distribution) that the individual with lowest income has always an income level greater than the minimum required to survive. If the difference between the minimum acceptable income level and the minimum needed to survive is lower than the value of the index, $R(g) > \hat{Y} - Y^s$, the income distribution, with dispersion of incomes with respect to the minimum acceptable income level equal to g, would not be acceptable for an egalitarian society with individuals behind the veil of ignorance and an income for each individual equal to the minimum acceptable income level.. If, instead, $R(g) < \hat{Y} - Y^s$, the income distribution would be acceptable and if $R(g) = \hat{Y} - Y^s$, the income distribution would just be acceptable.

3.3.- A simple numeric example

The aim of this paper is to see the possibilities of using the indexes of riskiness, based on the distribution of gains and losses in risky assets, to *order* income distributions. These indexes are based on the dispersion of the possible values with respect to the initial point. In the case of income distributions we have considered that the initial point is the “minimum acceptable income level”.

We have concluded above that the bigger the value of the index is, the worse the distribution will be. Once we have calculated the values of the indexes of the different distributions, we can conclude which distribution is better taking into account the *criteria* of these indexes.

Example 1:

Consider that we have two different income distributions, A and B, with minimum acceptable income equal to 5. In each distribution there are three possible income levels, all of them with the same probability to occur. The income levels of distribution A are 3, 7 and 8, each one with probability 1/3. In distribution B the values of the incomes are 2, 6 and 10, each one with probability 1/3.

Minimum acceptable income= 5				THE VALUE OF THE INDEXES	
				FOSTER-HART	AUMMAN-SERRANO
DISTRIBUTION A	3	7	8	2,7749	2,44595
DISTRIBUTION B	2	6	10	5,2803	5,08014

Both indexes are higher in the case of distribution B. So, distribution B is worse than distribution A, according to the *criteria* of the extensions of the indexes of Aumann-Serrano and with Foster-Hart.

These indexes do not only take into account the inequality of the distribution, but also the levels of the incomes. Given a minimum acceptable income level, the same inequality between incomes implies a lower value of any of the indexes with incomes at higher levels than with incomes at lower incomes. Two income distributions with the same inequality between incomes but with different levels of incomes can only have the same value of any of the indexes if the minimum acceptable income level of the two distributions is not the same. In particular, the difference between the minimum acceptable incomes must be such that the distribution of dispersions, with respect to the corresponding minimum acceptable income, is the same in both distributions.

3.4.- Comparisons with the index of Gini/Lorenz

There are several indexes that are used to compare income distributions. One of the most famous indexes is the index of Gini, associated to the Lorenz Curve. The index of Gini/Lorenz, as the indexes we are analyzing, is an index of total-order, because it can be used to compare any pair of different income distributions.

The extensions of the indexes of Aumann-Serrano and Foster-Hart are calculated taking into account the dispersion with respect to the minimum acceptable income level. Hence, it is important to know the value of this minimum acceptable income in the society or societies considered. Instead, the index of Gini is not referred to any minimum acceptable income level.

3.4.1- DIFFERENCE 1:

The first difference has to do with the dispersion between incomes. In the case of the index of Gini/Lorenz, if we change the levels of incomes and we keep the differences between incomes, the coefficient of the index changes:

Example 2:

				%POPULATION	%INCOME
DISTRIBUTION C	1	2	3	0,333333333	0,166666667
				0,666666667	0,5
				1	1
DISTRIBUTION D	101	102	103	0,333333333	0,330065359
				0,666666667	0,663398693
				1	1

In *example 2* we have two distributions, C and D. In both cases the dispersion between incomes is the same, but the levels of the incomes are different. Although the dispersion is the same, the value of the index changes, because in distribution C the 66.67% of the population has 50% of the income but in distribution D, the 66.67% of the population has 66.34% of the income. Taking into account the index of Gini, distribution D is more egalitarian.

In the case of our indexes, although the income levels change, the value of the index would not change if the dispersion with respect to the minimum acceptable income were the same.

Example 3:

minimum acceptable income C = 1.5	Yi	1	2	3
	gi	-0,5	0,5	1,5
minimum acceptable income D= 101.5	Yj	101	102	103
	gj	-0,5	0,5	1,5

In this example there are two income distribution (the same distributions that were used in the example 2), but the minimum acceptable income level is different in the two distributions (may be because each distribution belongs to a different country). This level is different in order to have the same dispersion of incomes with respect to the minimum acceptable income level in both cases. In this case, as this dispersion is equal in distribution C and D, so the value of the index of Aumann-Serrano is the same in both cases, and this also occurs with the index of Foster-Hart. However, if the minimum acceptable income level would be the same for both distributions the value of the index of Aumann-Serrano would be different for the two distributions, and this would also be true for the index of Foster-Hart.

3.4.2.- DIFFERENCE 2:

Our indexes do not only take into account the inequality between incomes, but also the levels of the income levels. That is why, if the income level of the highest income increases and the other incomes remain constant, the indexes decrease. So, if we take into account the criteria of these indexes, the last distribution is better.

Example 4:

minimum acceptable income level=5				THE VALUE OF THE INDEXES	
				FOSTER-HART	AUMMAN-SERRANO
E	1	8	10	6,5426	5,89808
F	1	8	12	5,6462	5,09839

In this example there are two different distributions, E and F, with the same minimum acceptable income level, 5. The two lowest income levels are equal in the two distributions, but the highest income is greater in F. If we compute our indexes in these distributions, we obtain that their value is lower in distribution F. So, these indexes do not only take into account the inequality of the distribution, but also the levels. In this example, the increase in the highest income compensates for the increase in income inequality.

But if we take into account the index of Gini, the inequality between incomes increases from E to F, so the index of Gini increases. That is why when we take into account the criteria of Gini, if the income level of the highest income increases maintaining the other income levels constant, the second income distribution is worse, because the inequality is bigger

4.- SOME RESULTS FOR THE EXTENSIONS OF THE INDEXES OF FOSTER-HART AND AUMANN-SERRANO

There are several results that have to be taken into account when we want to extend the riskiness indexes of Aumann-Serrano and Foster-Hart to the case of income distributions

4.1.-Result 1

These indexes do not only take into account the inequality of the distributions, but also the values that incomes can take in the future, because there may be an increase in the inequality but also an increase in the values of some incomes.

Example 4:

minimum acceptable income level=5				THE VALUE OF THE INDEXES	
				FOSTER-HART	AUMMAN-SERRANO
E	1	8	10	6,5426	5,89808
F	1	8	12	5,6462	5,09839

As we have analyzed above, the inequality increases from E to F, but in distribution F, everybody is equal or better than in E. So, the indexes improve (because they decrease). That is why we say that the indexes do not only measure the inequality, but also they take into account the income levels.

So, the indexes can allow for a greater inequality compensated with higher income levels (however, see Results 2 and 4 below).

4.2.- Result 2

Example 5:

minimum acceptable income level=10				THE VALUE OF THE INDEXES	
				FOSTER-HART	AUMMAN-SERRANO
distribution G	8	12	19	2,2345	2,09128
distribution H	5	12	19	8,0338	7,65828
distribution I	8	9	19	3,241	4,18
distribution J	8	12	16	2,3609	2,13939

In this example, we have reduced the level of one of the incomes in distributions H, I and J, with respect to distribution G, in the same amount. In distribution H the lowest income level has been reduced in 3. In distribution I the second lowest income level has been reduced in 3 and in distribution J the highest income level lowest income level has been reduced in 3.

We can see that when the change is made in the lowest income level, the indexes increase more than in the other cases. So, we can conclude that the indexes are more sensitive to changes in low incomes than to changes in high incomes.

4.3.-Result 3

What happens if we maintain the dispersion with respect to the minimum acceptable income level, but the minimum acceptable income level has changed? As we see in the following example the values of the indexes do not change.

Example 6:

distribution	minimum acceptable income level	incomes			FOSTER-HART	AUMMAN-SERRANO
K	5	2	6	10	5,2803	5,08014
L	10	7	11	15	5,2803	5,08014

In this example, the values of the indexes are equal in distributions K and L, because the dispersion of the incomes with respect to the minimum acceptable income level is the same. But we can see that the minimum acceptable income levels are different.

However, the values of the index of Foster-Hart for the two distributions are $R(g_K)=R(g_L)$, but we have that $R(g_K) > \hat{Y}_K - Y^S$ and $R(g_L) < \hat{Y}_L - Y^S$, where $\hat{Y}_K$ is the minimum acceptable income level of distribution K and $\hat{Y}_L$ is the minimum acceptable income level of distribution L . In this case the income distribution K would not be acceptable, while the income distribution L would be acceptable. So, a dispersion of incomes with respect to the minimum acceptable income can be accepted (in the sense of no-bancruptcy) with a high minimum acceptable income level but not-acceptable with a low minimum acceptable income level.

4.4.- Result 4

Example 7:

Minimum acceptable income=5				THE VALUE OF THE INDEXES	
distribution				FOSTER-HART	AUMMAN-SERRANO
M	2	20	1000	3,0015	2,73416
N	2	50	1000	3,0006	2,73072
O	2	500	1000	3,0001	2,73072
P	2	5000	10000	3	2,73072
Q	2	5000	100000	3	2,73072

Aumann and Serrano say that one of the bad things of the index of Foster-Hart is that the value of the index has to be at least the maximal loss of the income distribution.

Example 8:

minimum acceptable income level =5					
				index of Foster-Hart	index of Aumann-Serrano
distributions					
R	3	20	700	2,0016	1,82062
S	2	40	5000	3,00014	2,73062

In the case of Foster-Hart, the index of distribution R has to be at least 2 because its maximal loss is 2, and the index of distribution S has to be at least 3, because its maximal loss is 3. In distribution R the index of Foster-Hart is at least 2 in order to avoid bancruptcy and 3 in distribution S.

But in example 7, we can see that the index of Aumann-Serrano also has a threshold, and although we increase the gains of the individuals that are in a better situation, if the minimum income does not increase, the value of the index is almost constant.

So, in both indexes, if there are not changes in the income level of the individual with lower income, there is a level of per-capita income at which increases in the per-capita income do not improve the index, because the indexes have a minimum level conditioned to the maximal loss.

But this is an advantage of the indexes, because, although the individuals with high incomes improve their situation, the indexes take into account that those who are worse have not improved their situation.

4.5.- Result 5

The indexes may not order the income distributions in the same way, as they weight the income levels in each distribution differently.

Example 9:

Minimum acceptable income=10				index of		index of	
distribution				FOSTER-HART	order	AUMMAN-SERRANO	order
T	6	9	21	7,5255	1	8,49876	1
U	6	9	19	9,2802	3	10,1065	2
V	6	8	22	8,861	2	10,3177	3

In example 9, there are three different distributions, T, U and V, all of them with the same minimum acceptable income level equal to 10. The distributions are ordered differently by the two indexes. According to the extension of the index of Foster-Hart distribution V is better than distribution U, because the value of that index in the case of distribution V is lower. However, distribution U is better than V according to the extension of the index of Aumann-Serrano.

4.6.-Result 6

Example 10:

We will consider that there are two income distributions, g and h, both of them with two possible income levels. Each income level may occur with a probability of 0.5.

distributions	incomes	
W	8	20
X	7	23

Let us consider that there can be different minimum acceptable income levels. How does the ordering of the two distributions changes when the minimum acceptable income level changes?

		index of	index of	order of
MIL	distribution	FOSTER-HART	AUMMAN-SERRANO	preference
8,1	W	0,1008	0,14427	1
	X	1,1877	1,58706	2
9	W	1,1	1,4432	1
	X	2,3333	2,90225	2
10	W	2,5	2,95869	1
	X	3,69	4,5134	2
12	W	8	8,31235	1
	X	9,1667	9,62615	2
12,4	W	10,45	10,7062	1
	X	11,0077	11,4152	2
13	W	17,5	17,6642	2
	X	15	15,3218	1
13,5	W	35,75	35,833	2
	X	20,5833	20,8286	1
13,9	W	179,95	179,967	2
	X	28,5409	28,7224	1

When the minimum acceptable income level (MIL) is low, distribution W is preferred to distribution X, but when this income level increases, the ordering of income distributions changes. At low levels of the minimum acceptable income level, R(X)>R(W), so W is preferred, but when the minimum acceptable income level is close to the average income of the income distribution the order changes, R(W)>R(X). So, if we have to choose between two distributions the decision may depend on the minimum acceptable income level.

If the minimum acceptable income level changes, the order of preferences between income distributions can also change. So, the comparison, taking into account these indexes, depends on the minimum acceptable income level considered. But there may be a big discussion about the correct level of the minimum acceptable income.

This dependence on the minimum acceptable income level of the order of income distributions, taking into account the extensions of the indexes of Foster-Hart and Aumann-Serrano can look as a disadvantage. However, the relative losses and gains with respect to the minimum acceptable income level change when this level is modified, and the evaluation of the income distributions has to change.

5.- AN ILLUSTRATION OF THE EXTENSIONS OF THE INDEXES OF AUMANN-SERRANO AND FOSTER-HART: INCOME DISTRIBUTIONS IN SPAIN (1995 and 2002)

The last part of the paper is an application of these two indexes of riskiness to the case of income distributions. The data used in this illustration have been obtained from the web page of INE[3] (Instituto Nacional de Estadística). These data contain the income distributions of the years 1995 and 2002 in intervals of the minimum wage (SMI)[4]. These income distributions have been selected because the INE carried the "Encuesta Anual de Estructura Salarial" in those years[5].

It is considered that the minimum acceptable income level is twice the minimum wage (SMI) of the year 1995. In order to be able to compare the indexes of the years 1995 and 2002, we have to use the same minimum acceptable income level to compute the indexes. As we use the minimum acceptable income level of the year 1995 to calculate the index of 2002, the income levels of the year 2025 have to be deflated with the inflation (the inflation between these two yeas was 21,6%).

The minimum wage (SMI) of the year 1995 was 5275,2 Euros, so the minimum acceptable income level is 10550,4 Euros.

---

[3] www.ine.es
[4] SMI= minimum wage (*salario mínimo interprofesional*)
[5] Sara de la Rica suggested to use the data corresponding to those years

INTERVALS			INCOME OF REFERENCE	population percentage 1995	2002
For 0SMI to 1SMI	0	5275,2	2637,6	0,0842867	0,1308467
For 1SMI to 2SMI	5275,2	10550,4	7912,8	0,2656308	0,2744556
For 2SMI to 3SMI	10550,4	15825,6	13188	0,2653672	0,2609733
For 3SMI to 4SMI	15825,6	21100,8	18463,2	0,1692574	0,145543
For 4SMI to 5SMI	21100,8	26376	23738,4	0,0951873	0,0801788
For 5SMI to 6SMI	26376	31651,2	29013,6	0,0494951	0,0415529
For 6SMI to 7SMI	31651,2	36926,4	34288,8	0,0261815	0,0232352
For 7SMI to 8SMI	36926,4	42201,6	39564	0,0151868	0,0145214
For 8SMI to 9SMI	42201,6	47476,8	44839,2	0,009043	0,0095523
For 9SMI to 10SMI	47476,8	52752	50114,4	0,0063383	0,0056686
MORE THAN 10SMI	52752		55389,6	0,0140258	0,013472

In the year 1995, 34.99% of the workers had an income level lower than the minimum acceptable income level. In the year 2002, 40.53% of the workers had an income level lower than the minimum acceptable income level.

Taking into account the data of income distributions, the value of the indexes are the following:

	1995	2002
AUMANN-SERRANO	3805,6	5282
FOSTER-HART	8041,9	8938,9
GINI	0,3328	0,3579

The indexes of Aumann-Serrano and Foster-Hart indicate that the income distribution of the year 1995 is better than the income distribution of 2002, because the indexes are lower. The index of Gini indicates that the income distribution of 1995 is more egalitarian than the distribution of the year 2002.

In this particular case, with income distributions of the years 1995 and 2002, the indexes that we are analyzing order the income distributions in the same way as the index of Gini.

6.- REFERENCES

Aumann, R.J. and Serrano, R. (2008): "An Economic Index Of Riskiness", forthcoming in the "Journal of Political Economy"

Foster, D.P. and Hart, S. (2008): "An Operational Measure of Riskiness"