A simple model of betting in Basque pelota

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September 11, 2008

1 Introduction

Betting seems to have been associated with Basque pelota from its origin[1], at the beginning in the form of a pure bilateral basis, where a spectator would challenge anyone in the audience to exchange a particular bet. For instance: "10 duros that blue don't scores 12 points" or whatever prediction that could be taken as a bet by another spectator. Later the system evolved towards a centralized bilateral system, where the person to person betting takes place through middlemen who call the odds during the match in search of pairs of matching bettors ready to take either side of the bet at the odds called at that moment by middlemen.

In this work we provide a simplified model of the current betting system in Basque pelota matches that accounts for the favorite-longshot bias phenomena.

The favorite-longshot bias is the most discussed empirical regularity in betting. It means that betting odds are biased estimates of the probability of a team winning, longshots are overbet while favorites are underbet. That is, bettors value longshots more than expected given how rarely they win, and they value favorites too little given how often they actually win. Two broad sets of theories have been proposed to explain the favorite-longshot bias. First, neoclassical theory suggests that the prices that bettors are willing to pay for various gambles can be used to recover their utility function. That is, rational gamblers overbet longshots, which are the riskiest investment, due to risk love preferences. Thus, the neoclassical approach can reconcile both gambling and the longshot bias only by positing (at least locally) risk loving utility functions. Alternatively, the competing behavioral explanations emphasize the role of misperceptions of probabilities. Our simplified model accounts for the favorite-longshot bias from a neoclassical approach based on expected utility preferences[2].

In the model there are two types of bettors with the same information on the abilities of the players to win the match. The "cátedra", however, is assumed to be risk neutral, and the occasional bettors, risk lovers. This modelling allows us to derive the following results:

First, we derive the conditions under which occasional bettors are ready to accept a bet. Second, we find that our modelling always shows up the favorite-longshot bias[3].

[1] See Gonzalez (2005) for an antropological study of Basque pelota. For a review of economic analysis of betting in sports see Sauer (1998).

[2] For more detail about the favorite-longshot bias phenomena see Ates (2004, pp. 34-36) and Snowberg and Wolfers (2007).

[3] Shin (1991) distinguishes two types of bettors but with different information. He also shows that

And finally, for a continuum of types of risk lovers occasional bettors we derive the optimal odds set by the cátedra.

We are aware of the limitations of this model and we could extend it in several directions. We could introduce a finite number of individuals as the cátedra, instead of only one. We could also consider more middlemen in order to describe a competitive behavior among them to capture bets. But perhaps, the most interesting issue to study would be the dynamics of the betting system, that is, the possibility of bettors to hedge.

The work is organized as follows: in Section 2 there is a description of the Basque pelota matches and of the betting system. In Section 3 we specify a very simple model that tries to represent the betting system in Basque pelota matches. Section 4 concludes analyzing the distance from our modelling to the real situation and considers some further extensions in the future.

2 The Basque pelota betting system.

2.1 The game

As it is said in http://en.wikipedia.org/wiki/Basque_pelota:

> *Pelota* in Spanish, *pilota* in Basque and Catalan, or *pelote* in French is a name for a variety of court sports played with a ball using one's hand, a racket, a wooden bat, or a basket propulsor, against a wall (*frontón* in Spanish, *pilotaleku* or *pilota plaza* in Basque, *frontó* in Catalan) or, more traditionally, with two teams face to face separated by a line on the ground or a net. Their roots can be traced to the Greek and other ancient cultures, but in Europe they all derive from *jeu de paume*[4]. Today, Basque Pelota is widely played in several countries: in Spain and France, especially in the Basque Country and its neighbor areas. Also the sport is played in American countries like Perú, Mexico, Argentina and Uruguay.

the market always shows the longshot bias.

[4] Jeu de paume was originally a French predecessor of lawn tennis played without racquets. The players hit the ball with their hands, as in palla, volleyball, or certain varieties of pelota. Jeu de paume literally means: game of palm (of the hand). Even when bats, and finally racquets, became standard equipment for the game, the name did not change. It became known as "tennis" in English, and later "real tennis". (Source: http://en.wikipedia.org/wiki/Jeu_de_paume).

We focus on the sport that is played with bare hands, and hereafter it is called Basque pelota. Originally the Basque pelota was played face-to-face, but the introduction of more rapid rubber balls forced a change in the game: returning the ball to a wall instead of over the net. Nowadays the Basque pelota is played with a leather ball, which is handmade. This kind of pelota is played either singles or doubles. It is played by two teams: reds and blues play against each other by hitting a ball in turn against a wall on a court called a "frontón". The team that serves first is chosen by throwing a coin. When a team makes an error the opponent scores one point and serves to start the next point. The team that first reaches 22 points wins the match, which lasts an hour approximately[5].

2.2 A brief description of the betting system[6]

It is difficult to know which is the origin of betting, most probably it arose together with the game. Their rules were traditionally not written but passed orally and most probably this has been the main reason of the lack of economic studies, with the exception of the work by Llorente (2006) and Aizpurua and LLorente (2007), of this peculiar betting market. The system has endured the passing of time and nowadays its success is unquestionable. Observing it on live allows us to contemplate betting exchanges at different prices.

On Basque pelota matches a bet is described by two quantities: a quantity of money the bettor loses when he fails to predict the winning team and an amount of money the bettor wins if he guesses right. Bets can be made during the whole match, so while points are scored, odds change according to a rule. A bettor can place as many bets as he wants to, provided someone can be found to accept those bets. This means that bets are bilateral: a spectator chooses either the red team or the blue and wager an amount of money against another spectator that chooses the other team and wagers another amount of money (these amounts will be called the odds). The spectator who guesses right wins the money that her opponent loses minus 16% of it. This 16% is shared between organizers and middlemen. Each of them takes 8%. Middlemen make it possible the trade between bettors and work for the organizers. The money is paid and collected at the end of the match.

In this market there exists a group of regular bettors called la cátedra who jointly

[5]For more information see http://www.nabasque.org/NABO/Pilota.htm/ and Llorente (2006, p. 20).

[6]Some of the information in this subsection has been obtained from Llorente (pp. 20-23).

with middlemen decide (i) the team favorite to be the winner of the competition and (ii) the starting odds, which are announced prior to the beginning of the match. The members of this group bet large amounts of money along the entire match. At the beginning of the match, they always bet for favorite, but later in the match they usually hedge. There exists also another group of occasional bettors who often bet only once during the match. The members of this group bet relatively small amounts of money.

How are bets made? Several middlemen take up positions between the players and the audience, facing the latter, and start calling the odds. A bettor signals his wish to bet to the middleman and also indicates how many bets he wants to place. When the middleman sees the signal, calls the odds, looking for someone willing to accept the bet. When someone accepts the bet, the middleman prints two receipts - a red one and a blue one - on which the date, the odds and the number of bets are recorded. He places each receipt in a tennis ball and throws the red one to the person who bets for red and the blue one to the person who bets for blue. After each point the odds change depending on the new score**.** For example, if a middleman calls (100,60), he means that someone is ready to bet 100€ for the favorite team, and wants someone else to bet 60€ for the longshot[7].

At the end of the match people go to the middleman to pay to or to be paid. The loser must pay the amount shown on the receipt, but the winner is paid the money on the receipt minus the 16% commission taken by the organizers. The middleman must pay the winner even in the unusual but possible case that the loser has not paid.

Example: Suppose that the red team is the favorite one and that a middleman calls the odds (100,80). Then bettor 1 "signals" to the middleman that he is willing to bet for red and bettor 2 "signals" to the middleman that he is ready to bet for blue. The middleman sends a red ticket to bettor 1 and a blue ticket to bettor 2.

-100€	80€

bet for red

-80€	100€

bet for blue

This means that if a bettor bets for red and the red team wins the match, the bettor will win the 84% of 80€, but if red loses, he will lose 100€. And if a bettor bets for blue and the blue team wins, the bettor will win the 84% of 100€, but if blue loses, he will lose 80€.

[7] "Longshot" means non favorite.

3 The model

We propose a very simple model, which can be interpreted in two ways. The first one is to see it as a one-point match and the second one is to see it as a 22-point match in which bets are only allowed at the beginning of the match. In the first case, p (<0.5) would be the probability for the longshot to win the point and the match, while in the second, p would be the probability for the longshot to win the match.

In our simplified model a single privileged bettor, the *cátedra*, sets the odds and is risk indifferent. The bets offered by the cátedra can be taken by occasional bettors, who are risk lovers with different degree of risk love. Both cátedra and occasional bettors are assumed to be expected utility maximizers. It is also assumed that all bettors know the "true" probability of each player to win the match, p, and there is a middleman who calls the odds set by the cátedra and makes possible the trade[8] between the cátedra and occasional bettors. Since the role of the middleman is not significant in the model we will ignore the commission charged for the mediation.

For convenience we normalize odds such that $(1, m) : 0 < m \leq 1$ and hereafter m will be called the longshot's odd. In the sequel we assume the red to be the favorite team and the blue to be the longshot. Moreover it is assumed that the cátedra bets for red, the favorite, while occasional bettors for the longshot and can bet only once.

Now let us describe the bettors behavior.

3.1 Bettors behavior

We assume that all bettors, both the cátedra and occasional bettors, have expected utility or Von Neumann Morgenstern preferences.

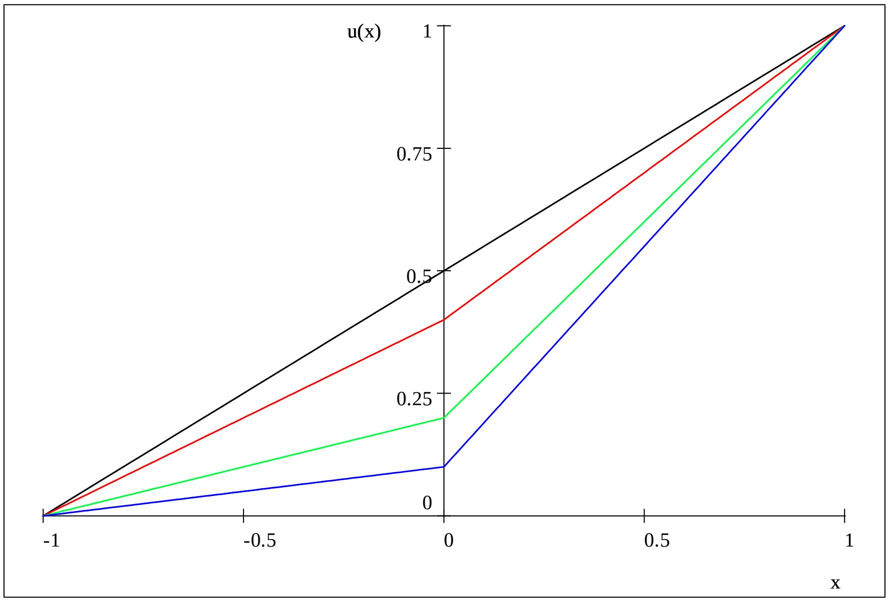
We start describing the occasional bettor's behavior. There is a continuum of types of risk lovers occasional bettors, one for each $\lambda \in \left(0, \frac{1}{2}\right)$. In particular, an occasional bettor of type λ's behavior is described as maximizing the expected utility for the following utility function on money (x):

$$u_\lambda(x) = \begin{cases} \lambda(x+1), & \text{if } \ -1 \leq x \leq 0 \\ (1-\lambda)x + \lambda, & \text{if } \ \ 0 < x \leq 1 \end{cases}$$

where $0 < \lambda < 1/2$.

Notice that if $\lambda = 1/2$ the individual is risk neutral and that the smaller λ, the more risk lovers would the occasional bettors be (see the following picture).

[8] In this work, "trade" means that an occasional bettor accepts a bet offered by the cátedra.



$\lambda = 0.5$ (black), $\lambda = 0.4$ (red), $\lambda = 0.2$ (green), $\lambda = 0.1$ (blue)

We assume that occasional bettors whose behavior is closer to risk indifference (i.e. of type λ close to 0.5) are more frequent than those whose behavior is more risky (i.e. of type λ close to 0). Formally, we assume the types are distributed according to a beta distribution with parameters $\alpha \geq 2$ and $\beta = 1$. This choice of parameters ensures a convex and "skewed to the right" distribution[9] given by

$$F(\lambda) = 2^{\alpha}\lambda^{\alpha}, \qquad 0 < \lambda < 1/2.$$

The corresponding beta density function is

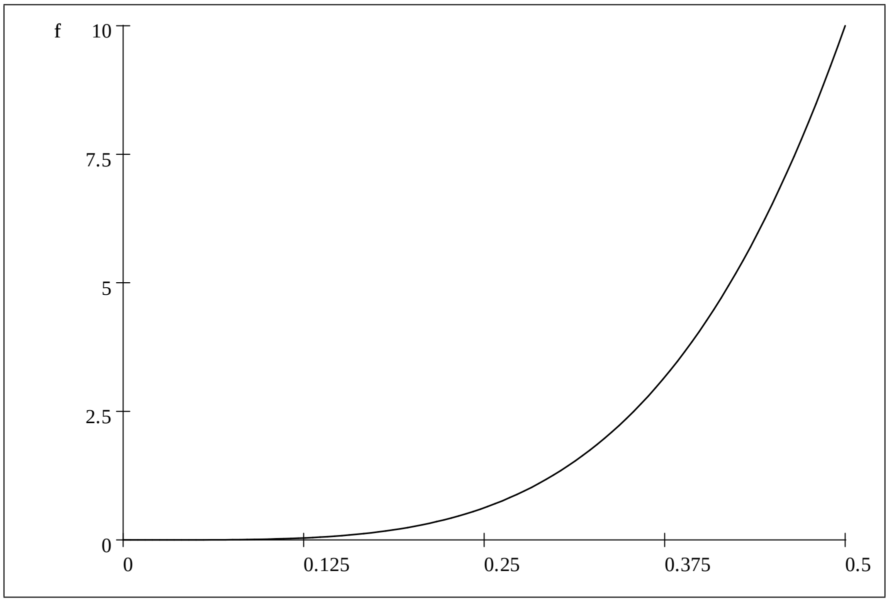
$$f(\lambda) = \alpha 2^{\alpha}\lambda^{\alpha-1}, \qquad 0 < \lambda < 1/2.$$

It is a straight line when $\alpha = 2$ and strictly convex when $\alpha > 2$. Although most of the results are basically similar for any value of $\alpha \geq 2$, we assume $\alpha = 5$ in order to provide numerical results. Thus, we assume that the density function is:

$$f(\lambda) = 160\lambda^4, \qquad 0 < \lambda < 1/2,$$

which can be represented as follows:

[9]The beta distribution has two shape parameters, α and β. When the two parameters are equal, the distribution is symmetrical. For example, when both α and β are equal to one, the distribution becomes uniform. If α is less than β, the distribution is skewed to the left. And if α is greater than β, the distribution is skewed to the right.



The corresponding c.d.f is:

$$F(\lambda) = 32\lambda^5, \qquad 0 < \lambda < 1/2.$$

As we have mentioned, we assume that occasional bettors can only accept one bet for blue. Each bet is represented as (-m, 1). This means that if the blue team wins the match the bettor will win 1 (notice that we are not considering the commission charged by the middleman in this simple model), and if the red team wins, the bettor will lose m.

Each occasional bettor is going to accept the bet set by the cátedra if the expected utility he obtains from the lottery that gives 1 with probability p and -m with probability $1 - p$ is greater than the utility he obtains from not betting. That is, the λ occasional bettor will accept a bet if

$$pu_\lambda(1) + (1-p)u_\lambda(-m) > u_\lambda(0)$$

Substituting the utility function and solving for λ we have that this condition holds whenever

$$p + (1-p)\lambda(-m+1) > \lambda$$

or equivalently,

$$\lambda < \frac{p}{1-(1-p)(1-m)}.$$

Notice that if $\frac{p}{1-(1-p)(1-m)} \geq \frac{1}{2}$ then all occasional bettors will accept the bet. It is very easy to check that the latter inequality holds if and only if $m \leq \frac{p}{1-p}$. Thus, have the following result:

Proposition 1 *If* $m \leq \frac{p}{1-p}$ *all occasional bettors will accept the bet, while if* $\frac{p}{1-p} < m \leq 1$*, then only those occasional bettors for which* $0 < \lambda < \frac{p}{1-(1-p)(1-m)}$ *will accept.*

If the odds called by the middleman are (1,m) with $0 < m \leq \frac{p}{1-p}$, then all occasional bettors will accept the bet, while if $\frac{p}{1-p} < m \leq 1$ then only a proportion of bettors, sufficiently risk lovers, will accept the bet, namely, those for which $0 < \lambda < \frac{p}{1-(1-p)(1-m)}$.

Let us illustrate these results with some numerical examples:

If $p = 0.2$ then $\frac{p}{1-p} = 0.25$, so that depending on m we have that:

-if $m \leq 0.25$ then all occasional bettors would bet,

-if $m > 0.25$ then only those that $0 < \lambda < \frac{p}{1-(1-p)(1-m)}$ would accept the bet. For instance,

if $m = 0.3$, then only those that $0 < \lambda < 0.45$,
if $m = 0.5$, then only those that $0 < \lambda < 0.33$,
if $m = 0.8$, then only those that $0 < \lambda < 0.24$.

If $p = 0.4$ then $\frac{p}{1-p} = 0.66$, so that depending on m we have that:

-if $m \leq 0.66$ then all occasional bettors would bet,

-if $m > 0.66$ then only those that $0 < \lambda < \frac{p}{1-(1-p)(1-m)}$ would accept the bet. For instance,

if $m = 0.7$, then only those that $0 < \lambda < 0.48$,
if $m = 0.8$, then only those that $0 < \lambda < 0.45$,
if $m = 0.9$, then only those that $0 < \lambda < 0.43$.

As expected, there is a relationship between m and p: the higher the true probability of the longshot to win the match, p, the greater the maximum value of the longshot's odd, m, at which all bettors accept the bet.

Now, we turn our attention to the cátedra's behavior. The cátedra is assumed to be risk neutral and can bet as many times as it wants. The bet that the cátedra is committed to exchange with an occasional bettor who accepts the bet $(1, m)$ can be represented by $(-1, m)$. Let $0 < \tau \leq 1$ be the mass of trades. Then, if the red team wins the match the cátedra will win τm, and if the blue team wins, the cátedra will lose τ. The cátedra would offer a bet if its expected profits are positive, that is,

$$[(1-p)m - p]\,\tau > 0$$

or equivalently,

$$m > \frac{p}{1-p}$$

We now address the following questions:

1. Under which conditions trade exists.

2. Given that the cátedra may choose the starting odds: which are the optimal odds for the cátedra?

3.2 Conditions under which trade of bets exist

As the cátedra would offer bets if and only if $m > \frac{p}{1-p}$, then in view of Proposition 1, we have the following

Proposition 2 *Trade between the cátedra and occasional bettors exists if and only if* $\frac{p}{1-p} < m \leq 1$.

This result can be interpreted as follows: trade exists if and only if m is in the interval $\left(\frac{p}{1-p}, 1\right]$. From Proposition 1, when the longshot's odd is higher than $\frac{p}{1-p}$, only those occasional bettors with a sufficiently risk loving attitude would accept the bet.

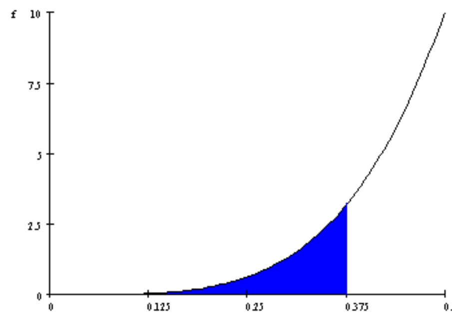
3.3 The optimal starting odds

The cátedra sets the starting odds in order to maximize its profits. To know the profits of the cátedra we need to know the proportion of occasional bettors who accept the bet for each possible m, that is, we need to know $\tau(m)$.

We have just seen that trade exists if and only if $\frac{p}{1-p} < m \leq 1$, therefore from Proposition 1 we know that occasional bettors who can accept a bet are those with λ in the interval $0 < \lambda < \frac{p}{1-(1-p)(1-m)}$. To derive $\tau(m)$ recall that types (λ) are distributed according to a beta function given by $F(\lambda) = 32\lambda^5$, $0 < \lambda < 1/2$. Therefore, for a given m:

$$\tau(m) = \Pr\left(\lambda < \frac{p}{1-(1-p)(1-m)}\right)$$

$$= F\left(\frac{p}{1-(1-p)(1-m)}\right) = 32\left(\frac{p}{1-(1-p)(1-m)}\right)^5.$$



In the above picture, the shadowed area represents $\tau(m)$ assuming that

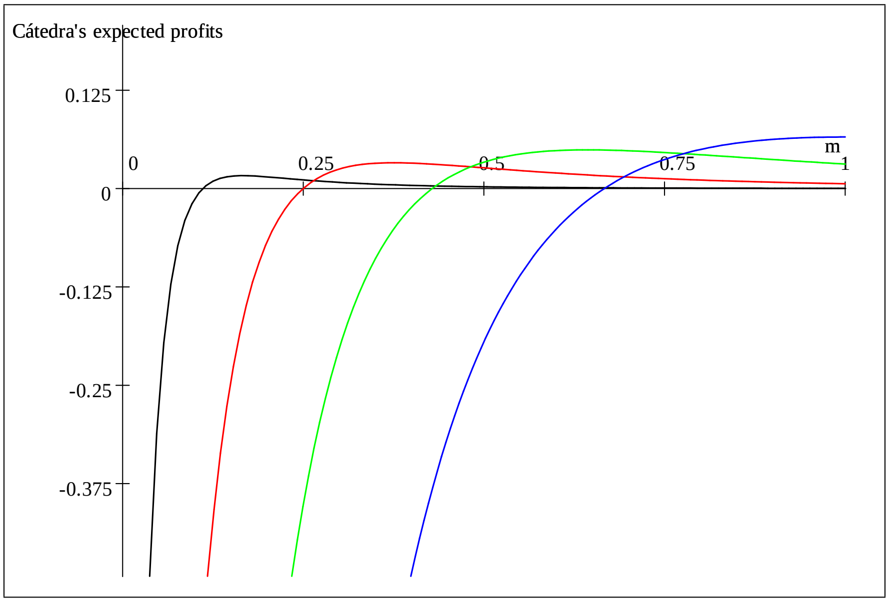
$$\lambda = \frac{p}{1-(1-p)(1-m)}.$$

In order to maximize its expected utility the cátedra must solve the following problem:

$$\max_{m} \pi(p,m) = [(1-p)\,m - p]\,32\left(\frac{p}{1-(1-p)(1-m)}\right)^5$$

$$\text{s.t. } \frac{p}{1-p} < m \leq 1$$

Plotting the functions that we want to maximize, which are the cátedra's expected profits, for different p's we obtain the following:



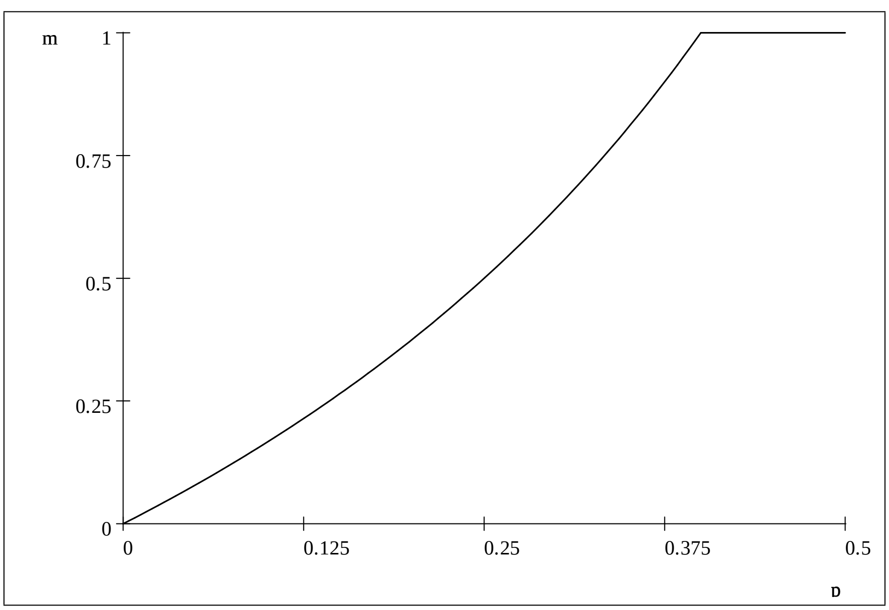
p=0.1 (black), p=0.2 (red), p=0.3 (green), p=0.4 (blue)

Solving this maximization problem we obtain the following

Proposition 3 *The optimal starting odds set by the cátedra are* $(1, m^*)$*, where* m^* *is as follows:*

-if $0 < p \leq 0.4$*, then* $m^* = \frac{3}{2}\frac{p}{1-p}$*,*

-if $0.4 < p < 0.5$*, then* $m^* = 1$*.*



Optimal longshot's odds for different p's

We can say that the starting odds are not consistent with the "true" probabilities since $m^* = \frac{3}{2}\frac{p}{1-p} > \frac{p}{1-p}$ when $0 < p \leq 0.4$ and $m^* = 1 > \frac{p}{1-p}$ when $0.4 < p < 0.5$. In other words, the betting market so modelled shows the so called longshot bias.

Let us illustrate the result with some numerical examples. For p in the interval $0 < p \leq 0.4$, we have the following table:

p	m^*	$\pi(p, m^*)$
0.0625	0.1	0.01024
0.117647	0.2	0.0192753
0.166667	0.3	0.0273067
0.210526	0.4	0.0344926
0.25	0.5	0.04096
0.285714	0.6	0.0468114
0.318182	0.7	0.0521309
0.347826	0.8	0.0569878
0.375	0.9	0.06144

And for p in the interval $0.4 < p < 0.5$, we have the following values:

p	m^*	$\pi(p, m^*)$
0.4	1	0.065536
0.42	1	0.0669139
0.45	1	0.059049
0.47	1	0.0440342
0.49	1	0.0180784

As can be observed, in the first interval it seems clear that the cátedra's expected profits increase with the probability for the longshot to win, while in the second interval the cátedra's expected profits first increase, but then decrease after a certain value of p until they reach zero-profits for $p = 0.5$. Let analyze this point with more detail.

The cátedra's expected profits are

$$\pi(p, m) = [(1-p)m - p]32\left(\frac{p}{1-(1-p)(1-m)}\right)^5.$$

We can distinguish two cases:

First, when $0 < p \leq 0.4$, from Proposition 3 we obtain that

$$\pi(p, m^*) = \frac{2^9}{5^5}p,$$

and since the partial derivative of $\pi(p, m^*)$ with respect to p is positive for all p, we conclude that the cátedra's expected profits increase with p, whenever p is in the interval $(0, 0.4]$.

Let us analyze now the case in which $0.4 < p < 0.5$. From Proposition 3 we have that

$$\pi(p, m^*) = 32p^5(1 - 2p).$$

The partial derivative of this function is

$$\frac{\partial \pi(p, m^*)}{\partial p} = 32p^4(5 - 12p),$$

Thus, $\pi(p, m^*)$ reaches a maximum when $p = \frac{5}{12}$. It is easy to see that the cátedra's expected profits increase with p if $0.4 \leq p < \frac{5}{12}$, while if $\frac{5}{12} < p < 0.5$, then they decrease until they reach zero at $p = 0.5$. This can be explained because there are two opposite effects affecting the cátedra's expected profits. Since $m^* = 1$, starting odds are $(1, 1)$, so that the higher the p is, the higher the proportion of occasional bettors who accept the bet is, and therefore the higher τ is. On the other hand, as p increases, the cátedra's expected profits for each bet accepted decrease. The trade-off between this two effects causes a change of tendency at $p = \frac{5}{12}$. So, ignoring this last extreme case when $m^* = 1$, we can conclude that the cátedra prefers to have competing two players with similar abilities to a too unbalanced match.

4 Critique of the model and further extensions

In this work we have specified a very simple model and we are aware of its limitations. Let us make some remarks that evidence the distance from the model to the real situation.

1. *The match.* Our model can be interpreted either as a one-point match or a 22-point match in which bets and probabilities are taken into consideration only before the match starts. In a real 22-point match things are much more complicated. The odds change along the match according to a pre-established rule and bets are possible at any point of the match. In particular, this opens the possibility to analyze the dynamics of the betting system and the possibility of *hedging*, which is often seen in the *frontón*. To deal with a model of a dynamic match, one must consider a probabilistic model based on the probabilities of scoring each point (a more realistic and complicated model should assign different

probabilities depending on which player serves the point). The assumption that the probabilities of scoring a point are objective and common knowledge is a strong idealization. Also it would be interesting to study the properties of the rule according to which odds change along the match depending on the score.

2. *The cátedra*. In our model the cátedra is represented by a single risk-indifferent player who sets the odds. Actually, the cátedra consists of a group of regular bettors whose risk-indifference is not well-established and whose relationship with the middlemen is not completely transparent. In particular, their role (joint with middlemen) in the setting of the starting odds is not completely clear.

3. *The middleman*. In our model middlemen are practically ignored: there is just one middleman who calls the odds set by the expert. In real standard matches there are usually 9 middlemen and there may up to 18 in a final competition. They live on their job, receiving a commission of 8% of the amount won by every winning bettor. Their role is not only to match bets, since it is known that they often bet. Middlemen also have a degree of autonomy to make betting decisions with the cátedra money. It would be interesting to study this point and take middlemen into account as a separated players. The existence of competitive behavior among middlemen to capture bets could be also an interesting question to be taken into consideration.

4. *Occasional bettors*. In our model, occasional bettors can accept a bet only once. Actually, they can accept different bets in different moments of the match. In the model we have considered a set of occasional bettors who follow a specific beta distribution. It would be interesting to study if there is a set of distributions with which we can derive similar results. Other possible extension could be to consider different functional forms for the occasional bettors' utility function. Thus instead of consider a linear utility function, a quadratic one could be considered (since the occasional bettors are risk lovers, the quadratic utility function should be convex) and see how the results change.

References

[1] Ates, C. O. 2004. "Behavioural Finance and Sports Betting Markets." Master thesis. Aarhus School of Business, University of Aarhus.

[2] González Abrisketa, O. 2005. *Pelota Vasca. Un ritual, una estética.* Muelle de Uribitarte, Eds. Bilbao.

[3] Llorente, L. 2006. "Bets, Wagering Markets and Simple Theoretical Benchmarks that support their existence: The case of the Pelota Betting System in Navarra: Bayesian learning and efficiency." Doctoral Thesis, Universidad Pública de Navarra.

[4] Sauer, R. D. 1998. "The Economics of Wagering Markets." *Journal of Economic Literature* XXXVI: 2021-2064.

[5] Shin, H. S. 1991. "Optimal Betting Odds against Insider Traders." *The Economic Journal* 101: 1179-1185.

[6] Snowberg, E. and J.Wolfers. 2007. "Explaining the Favorite-Longshot Bias: Is it Risk-Love or Misperceptions?" Mimeo.