Remaining Poor or Growing at a High Rate: a Sectoral Analysis

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Abstract. This paper shows how affects the productivity growth in each sector to the evolution of the aggregate productivity and the income per capita of the poorest countries. Given the sectoral data available, the analysis is based on 9 countries for the period $1975-2010$. In order to determine the sectors leading their evolution, we construct a model that allows us to answer what would have happened if they had followed a distinct sectoral productivity path. We find that agriculture and personal and government services are sectors influencing greatly in the relative aggregate productivity and income per capita growth of countries. In addition, we observe that the productivity growth in manufacturing has been important for the countries that moved either to a higher quintile or became poorer. However, the productivity growth in trade services was relevant for the countries that remained in the lowest quintile or fell to it. In general, experiencing the sectoral productivity path of countries with a higher increase in income per capita, would have increased the relative aggregate productivity and income per capita of countries with respect to that observed.
Keywords. Income per capita, productivity, quintile.

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

Most countries have experienced an increase in their income per capita from the 1970's until nowadays (table 3.1).

The analysis of Park and Mercado (2017) presents which are the main factors that have allowed countries to move towards higher quintiles of the income per capita distribution, given their initial level. In addition, the authors try to answer whether the determinants of economic growth are the same or vary over time. Their analysis is based on a sample of 182 countries from the year 1964 to 2010's. In order to find which factors enable countries to move to a higher quintile, they construct probits with the potential determinants of growth as the independent variables. The accumulation of capital, both human and physical, as well as the discovery of oil resources are the main determinants in countries' economic growth. However, the authors found that the relevance of these factors vary not only over time but also depending on the income of countries. Indeed, there exist other factors like population growth, inflation or civil liberties that played an important role in certain periods of time.

McMillan et al. (2017) show which is the role played by the structural change and the sectoral productivity growth on the aggregate labor productivity evolution of 7 developing countries (Botswana, Ghana, Nigeria, Zambia, India, Vietnam and Brazil). By decomposing the expression for aggregate labor productivity, they try to find how much of the changes in this variable can be attributed to the structural change and the within sectoral productivity evolution. Through this analysis it can be observed that there existed great differences in the structural changes of these 7 developing countries.

In Costa et al. (2016a) and Costa et al. (2016b), the authors classify countries depending on their stage of development, given their income per capita growth rate and their position with respect to the economic leader. In order to show the factors that allow countries to grow and reach the economic leader, the USA, the authors examine the evolution of Japan, China and Mexico. The authors claim that the development of countries is driven by a continuous growth in productivity, which is mainly due to the ability of countries to implement the advances in technology and managerial practices. In these analysis, authors find that institutions play an important role in the development of countries. Particularly, they observe that more opened institutions allow countries to grow over larger periods of time.

Mamo et al. (2017) analyze how affected the mining sector to the development of countries. These authors use evidence from 3, 635 districts of 42 Sub-Saharan African countries for the period $1992 - 2012$. In this analysis, the development of a district is measured through the presence of night-time lights in it. The main finding in this article is that mining discoveries constitute an opportunity for poor countries to develop. However, the only effects of a mining discovery on the income per capita of a country are momentary since it is fulfilled the “resource curse” view.

The analysis of Duarte and Restuccia (2010) aims to answer if differences in the sectoral productivity path among countries produce the observed differences in their structural changes. These authors elaborate a general equilibrium model in order to perform some replications indicating the relevance of each sectoral productivity growth. For this purpose, they construct a panel data including 29 countries from 1956 to 2004. As a reference, the authors use the USA. This analysis shows that structural transformation can initially increase aggregate productivity, but when countries specialize in the services sector, their aggregate productivity decrease. In addition, the authors claim that sectoral productivity differences among countries, in levels and growth, explain the broad differences in their structural transformation and aggregate productivity paths.

In 2017, Üngör (2017) modified the general equilibrium model of Duarte and Restuccia (2010) disaggregating the industry sector in two sectors and the services sector in six sectors. This variation in the model supposed an improvement in its estimations. Particularly, this analysis focuses on the role of sectoral productivity in the evolution of 11 Asian and Latin American countries from 1963 to 2010. One finding of this analysis is that the stagnation of Latin American countries is due to the poor growth of productivity in most sectors, rather than in a specific sector. The second finding of the analysis is that the manufacturing and wholesale sectors have a great positive influence in the aggregate productivity evolution of countries.

Our analysis tries to see whether the differences in the sectoral productivity paths followed by the poorest countries can explain the differences observed in their aggregate productivity and income per capita evolution. For this purpose, we use a model following the one elaborated by Duarte and Restuccia (2010) and the later modifications of Üngör (2017). In contrast to the original model of Duarte and Restuccia (2010), we consider that the economy can be decomposed in 9 different sectors, like Üngör (2017). This will allow the model to capture better the labor reallocation and the productivity growth of countries. Another important feature of our model that distinguishes it from the Üngör’s model is the implementation of the utility function similar to the one used originally by Duarte and Restuccia (2010). The utility function of Üngör (2017) assumes that individuals consume only the amount of agricultural goods needed to survive, whereas the utility function of Duarte and Restuccia (2010) allows individuals to consume more agricultural goods than the necessary to survive. This model will enable us to obtain the aggregate productivity and income per capita paths of countries, taking into account the performance in their sectoral productivities and that countries may produce an amount of agricultural good higher than the required to survive. Moreover, it will allow us to obtain the evolution of the aggregate productivity and income per capita of countries if they had experienced different sectoral productivity paths. Therefore, we will be able to evaluate the contribution of the productivity growth in each sector to the evolution of the aggregate productivity and income per capita of countries. Given the interest in the lowest quintile and the lack of its sectoral information, our analysis is based on 9 countries for a period of 25 years, from 1975 to 2010.

The analysis will follow this structure:

In the second section, we present the steps taken to perform our analysis and describe the data that will be used for the initial analysis. In the following section are presented some observed facts regarding the evolution of the income per capita and aggregate productivity of the countries analyzed. The fourth section contains the description of the model we use. The fifth section explains how will be done the calibration of the model. The next section evaluates the performance of the model by comparing its estimations with the data. After this, we present the results obtained through the experiments. Finally, we review the main findings of our analysis and present several lines for further research.

Chapter 2

Description of data

The aim of this analysis is to study whether the paths in sectoral productivity experienced by some of the poorest countries in 1975, determined their sectoral labor allocation and if so, what is its relevance in the aggregate productivity and on their income per capita. Particularly, we focus on the countries that were among the lowest quintile of the income per capita distribution in 1975 and in 2010, in other words, that were among the 20% poorest countries at the beginning and at the end of the period. Unfortunately, the analysis is restricted *only* to the cases for which there is sectoral data available.

In the sample we use, although there are few observations, we can distinguish clearly different patterns in the income per capita evolution. Some of the countries in the sample were initially among the poorest countries in terms of income per capita, but did a good performance and were able to overcome poverty before 2010. Several countries that started also among the poorest, remained being poor. The remaining countries we analyze began the period in a higher quintile, but were not able to grow enough to maintain their position and became part of the poorest quintile by 2010. These differences let us analyze whether an specific allocation of resources has an impact on the aggregate productivity and the income per capita.

Firstly, we will get a general view, looking at the evolution of the GDP per capita and the aggregate productivity of countries, paying special attention to the countries in our sample. In addition, we present also the evolution of the sectoral productivity and labor allocation for these countries.

In this analysis, we will consider not only countries as individual cases, but we will gather them according to their income per capita evolution in three groups. This will enrich our analysis because it will allow us to see if countries with similar income per capita trends present some specific characteristics that differentiate them from the other groups.

Secondly, we will present the model that will help us analyze whether changes in sectoral productivity lead to sectoral labor reallocation affecting to the aggregate productivity of countries. We will calibrate the model using as a reference the US economy. Results from this model will be presented and compared to what it is observed in the data. This comparison will be useful to evaluate the performance of the model in predicting the sectoral labor reallocation. The higher the accuracy of the model, the more reliable would be the experiments done with it.

The analysis concludes analyzing what would have happened to the evolution of the relative aggregate productivity with respect to the USA and the income per capita of some countries if instead of having their sectoral productivity had had the sectoral productivity of others. Particularly, we would like to see how would have been the evolution of the sectoral allocation and the aggregate productivity of countries that remained or entered in the lowest quintile by 2010 if they had experienced the sectoral productivity of the countries that moved to a higher quintile. Moreover, it would be interesting to analyze what would have happened to the evolution of the aggregate productivity of countries that moved to a higher quintile if they had experienced some different sectoral productivity evolution. These analysis can show the relevance of the sectoral productivity in the aggregate productivity of countries.

2.1 Data

The data used in this analysis comes from two different data sets depending on the final purpose. If aggregate data is required, we use the Penn World Tables version 9.0 [1](Feenstra et al., 2015), whereas if the analysis is done at the sectoral level, we rely on the sectoral information contained in the Groningen Growth and Development Centre (GGDC) 10-Sector database[2](Timmer et al., 2015).

The Penn World Tables version 9.0 (from this moment PWT) gives information about aggregates of income, inputs and productivity of countries. We can find variables such as the real and current gross domestic product and the population engaged (labor) for 182 countries over the period $1950 - 2014$.

The income per capita of countries is the variable that we use to classify poor and rich countries, and see how much they have improved, in economic terms, over the period studied. The income per capita of countries is measured as the Gross Domestic Product (GDP) per capita in Purchasing Power Parity terms (PPP). We obtain the GDP per capita in PPP terms dividing the real GDP in PPP terms, from the PWT, by the population in each country from the same database. We consider the GDP in real terms to avoid possible biases produced by the inflation of countries. Using the income in PPP terms allows to compare the data from different countries because it establishes a common set of international prices.

We will use this variable to analyze how was the income per capita distribution in 1975 and how it has changed over the period studied. In addition, it will allow us to see which countries are in the extremes of the initial and final distributions, and how was the evolution of their income per capita over the period.

For the average annual growth rate of the income per capita, we will use the initial and final income per capita of countries calculated as the GDP of countries from the national accounts divided by their population ($GDPpcna$). Particularly, the average annual growth rate of the income per capita of country i, expressed as a percentage, has been computed as follows:

$$\left(\left(\frac{GDPpcna_{i,2010}}{GDPpcna_{i,1975}}\right)^{\frac{1}{2010-1975}} - 1\right) \cdot 100.$$

Other important variable of the analysis is the productivity, in aggregate and in sectoral terms. We understand aggregate productivity as the production per worker of one country. Therefore, it is measured in terms of the real GDP in PPP of a country divided by the number of people engaged. Both variables are obtained from the PWT data set. There is an exception: in the case of Botswana, from 1975 to 1979, there is no data on the people engaged. Since the average annual growth of the production per worker for $1980 - 2010$ from the PWT and the GGDC databases are

[1] Available at http://www.rug.nl/ggdc/productivity/pwt/
[2] Available at https://www.rug.nl/ggdc/productivity/10-sector/

similar (1.039 with the PWT and 1.032 with the Groningen database), we will use the evolution of this variable observed with the Groningen database to estimate the aggregate productivity from $1975-1979$ given its value for 1980.

The GGDC 10-Sector database considers 10 sectors in the economy: agriculture (AGR), mining (MIN), manufacturing (MAN), utilities (PU), construction (CON), trade services (WRT), transport services (TRA), business services (FIRE), government services (GOV) and personal services (OTH). It provides information on the value added (current and constant in domestic prices) and the employment in each of the previous activities for 42 countries. Since the data for Egypt considers jointly the government and the personal services (OTH2), as in Üngör (2017), we have gathered these sectors for all the countries and years. Therefore, the sectoral decomposition extends to a total of 9 sectors. Another inconvenient is that the period of available information changes among countries. Although there are countries with available data from the $1950's$, there are others for which there is no data until the $1970's$. Therefore, we have considered the more restricted case to include the greatest amount of countries possible, which goes from 1975 until 2010.

In order to homogenize the analysis we will analyze the aggregate and the sectoral productivity data for the period $1975-2010$.

The aim of this paper is to analyze only the performance of poor countries. Given the lack of sectoral information, we study only 9 countries: Botswana (BWA), China (CHN), Egypt (EGY), Ethiopia (ETH), India (IND), Kenya (KEN), Malawi (MWI), Senegal (SEN) and Tanzania (TZA). Almost half of them were poor in 1975, but could improve their income per capita noticeably with respect to the other countries of the sample by 2010 (Botswana, China, Egypt and India). Two of them did not perform so successfully and remained being among the poorest countries in 2010 (Ethiopia and Malawi). The remaining countries of the sample (Kenya, Senegal and Tanzania) were not in the lowest quintile of the income per capita distribution in 1975 but became by 2010.

The *sectoral productivity* refers to the value added per worker generated by each sector. Given that the sectoral data is not measured in PPP terms, we cannot compare it directly between countries. Therefore, we have converted it following Duarte and Restuccia (2010) and Üngör (2017) in order to have sectoral information that can be compared between countries. The idea of these authors is to calculate the constant value added in PPP terms generated by each sector divided by the number of employees that produced it. The method they use so as to compute the sectoral productivity is

$$\frac{\frac{Value\,added\,sector\,i}{Total\,value\,added}\cdot GDP\,in\,PPP}{Employment_i}\; i=a,1,\ldots,8,$$

where the GDP in PPP terms comes from the PWT database and the remaining variables used for its computation from the Groningen data set. The inconvenient of this method is that it assumes that the relative prices across sectors remain constant. However, it has been found that the sectoral PPP-conversion differs over time (Duarte and Restuccia, 2010).

The sectoral labor allocation is a relevant characteristic of countries that affects their aggregate productivity and indicates their degree of development, given their stage in the structural transformation (Duarte and Restuccia, 2010). The *sectoral labor allocation* shows the percentage of the total labor force that is allocated to each sector.

Chapter 3

Observed facts

Table 3.1: Bounds and range of the quintiles

	1975	2010
Percent	20	
Min	279.99	556.56
Max	1577.36	2563.13
Max/Min	5.63	4.61
Percent	40	
Min	1602.10	2629.2
Max	2797.90	7423.71
Max/Min	1.75	2.82
Percent	60	
Min	2890.86	7478.05
Max	5980.52	15047.62
Max/Min	2.07	2.01
Percent	80	
Min	6207.90	15419.46
Max	12902.33	34770.87
Max/Min	2.08	2.25
Ratio $80/20$	8.18	13.57

Source: Own elaboration from the PWT

Each of the percentages on table 3.1 represents a quintile of the income per capita distribution. The 20 percent of the poorest countries, which are at the bottom of the income per capita distribution, represent the lowest or first quintile. The 40, 60 and 80 percentages represent the second, third and fourth quintiles respectively. We can observe that the income per capita has increased from 1975 to 2010 in all the quintiles of the income per capita distribution.
However, the behavior of the $80/20$ ratio reflects that the income per capita has not grown the same in all groups. This ratio indicates the difference in the income per capita of rich countries in comparison to the income per capita of the poor ones. Its increase implies that the income per capita of the rich countries has grown much more than the income of the poor ones.

Table 3.1 shows that there are not only differences between distinct quintiles, but that the income per capita also varies within quintiles. This table presents the maximum and the minimum income per capita within each quintile. It is noticeable that there exists a greater difference between the countries in the lowest quintile than between the countries in the highest quintile. The evolution of this ratio shows that the difference between the maximum and minimum income per capita within a quintile has declined for the lowest quintile. On the contrary, this difference has augmented for the second quintile, whereas it has almost remained stable for the highest quintiles during the 35 years analyzed.

Table 3.2: Poorest countries

1975		2010	
GDP per capita (in PPP)	Country	GDP per capita (in PPP)	Country
1 quintile		1 quintile	
596.49	Ethiopia	1039.27	Ethiopia
1219.81	Egypt	1043.87	Malawi
1298.84	Malawi	2002.3	Tanzania
1372.13	China	2122.80	Senegal
1375.88	Botswana	2552.27	Kenya
1400.81	India		
2 quintile		2 quintile	
1650.49	Senegal	4386.10	India
1894.83	Kenya		
1984.25	Tanzania		
		3 quintile	
		9389.47	China
		9406.38	Egypt
		12107.16	Botswana

Source: Own elaboration from PWT

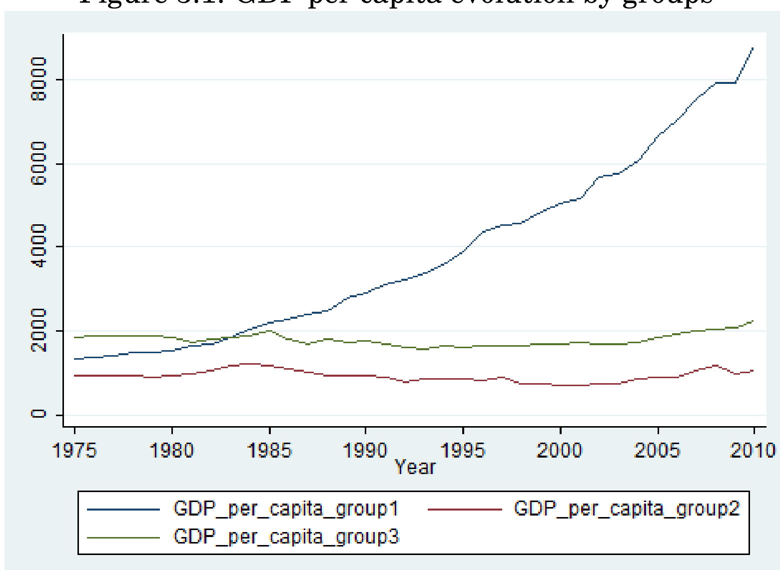
As we have seen so far there is a great variety in the levels and the evolution of the income per capita of countries. This analysis focuses on the lowest quintile of the income per capita distribution in order to study why some countries were able to grow faster and move to a higher quintile while others remained among the countries with lowest incomes. We focus our attention in the lowest quintile because of the greater difference in income per capita between its members, the richest country has an income per capita 5 times higher the income of the poorest country. Given this notorious gap, it would be interesting to analyze the factors that allow some countries at the bottom or in the middle of the lowest quintile to surpass the income per capita of the richest countries in that quintile and move to a higher one, whereas other members of the lowest quintile remain in it. Table 3.2 contains the countries among this group in 1975 and 2010, for which there are sectoral data available. Countries are colored depending on their income per capita evolution: they are blue if they were able to move to a higher quintile (group 1), red if they remained in the lowest quintile (group 2) and green if although starting in a higher percentile, they ended in the lowest one (group 3). By the side of each country is presented their income per capita.

If we make a deeper analysis, we can see that within the first quintile there were three levels initially: Ethiopia marked the lowest one, Egypt and Malawi were above it, but did not reach the income of China, Botswana and India. Looking at the income per capita evolution of these countries, we can observe several facts that *a priori* could be unexpected, given their level of initial income per capita.

Egypt and Malawi had very similar income per capita in 1975, however, Egypt was able to grow at a very high rate and end up in the third quintile by 2010, whereas Malawi remained to be very poor. Can we explain these different patterns by their corresponding sectoral characteristics? What would have happened to Malawi if it had experienced the same changes as Egypt? What would have happened to Malawi if it had experienced the same sectoral productivity paths as in Ethiopia?
Another interesting fact is the evolution of India with respect to the other members of the first group. Why India, that was the country with the highest income per capita within the first quintile in 1975, ended the period in the second quintile while the other members of the first group finished the period in the third quintile?

The case of Tanzania is remarkable because although having the highest initial income per capita of the third group, it ended having the lowest income per capita of this group by 2010. What was the difference between this country and the other members of the third group? In general, what was the difference between the third and the first groups?

Figure 3.1: GDP per capita evolution by groups



Source: Own elaboration from PWT

In figure 3.1, differences in the income per capita evolution among groups can be clearly appreciated. It is remarkable the increase in the income per capita experienced by countries of group 1 from the late 1970's. This growth allowed the first group surpassing the income per capita of the third group by the early 1980's. Looking initial and the final years, we can observe that the income per capita of groups 2 and 3 has increased slightly. However, there has not been a steady growth. Instead, the income per capita of groups 2 and 3 experienced a positive evolution until 1985, followed by negative growth until 2005. After this year, economies in these groups seemed to recover, but between 2008 and 2009 they suffered a downturn from which they were able to recover by 2010.

Table 3.3 indicates the average annual growth rates of the income per capita for each country and group from 1975 to 2010. In this table we can see that only 3 of the 9 countries grew at a rate lower than 1%, which is interpreted as the threshold to be considered in the Malthussian trap (Costa et al., 2016a). Two of the countries inside the Malthussian trap were part of group

Table 3.3: Average annual growth rate of the income per capita

Country	Average annual growth (%)
BWA	4.88
CHN	5.92
EGY	4.03
IND	3.81
Mean	**4.66**
ETH	1.08
MWI	0.53
Mean	**0.80**
KEN	0.68
SEN	0.25
TZA	1.15
Mean	**0.70**

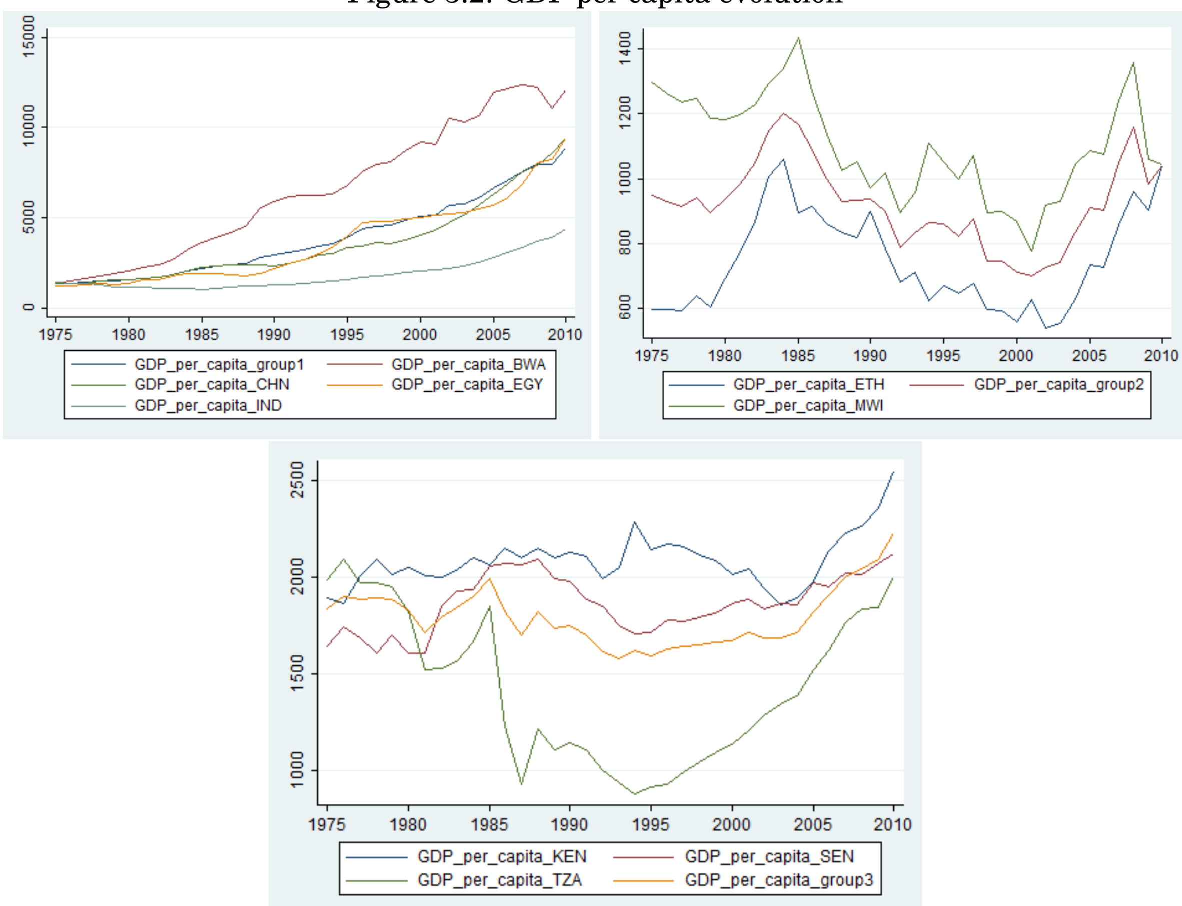
Source: Own elaboration from PWT

3 (Kenya and Senegal), while the other one was from the second group (Malawi). The remaining countries of groups 2 and 3 grew at a rate slightly higher than 1%. Countries of group 1, however, experienced high annual growth rates ranging from almost 4% to 6%.

On average, we can observe that group 1 grew at a rate of about a 4.5 percent each year. On the contrary, groups 2 and 3 grew less than 1 percent each year.

Differences within groups are shown in figure 3.2. The income per capita evolution of the countries in the first group is the most alike. China and Egypt grew around the mean, whereas Botswana and India were far from it. In the case of Botswana, the income per capita of the country started being above the other members of the group by the early 1980's. From that moment, the gap between Botswana and the other members of group 1 increased. On the contrary, India differentiates from its peers because its income per capita evolution was much slower. In fact, it is not until the second half of the 80's decade, that a continuous positive growth is appreciated.
Concerning the second group, it is remarkable that although the initial income per capita of Malawi was much higher than the one of Ethiopia, their final incomes per capita are very similar. Initially, we can observe a great difference in the level of income per capita of both countries. This gap seemed to reduce until the early 1980's due to the increase in Ethiopia's income, but it remained steady until the latest years, when the income per capita of Malawi decreased abruptly while the income per capita of Ethiopia kept growing. Nevertheless, their paths are full of sharp increases and decreases of the income per capita.
In group 3, there exists more diversity in the income per capita evolution. Initially, Tanzania was the country with the highest income per capita, but it experienced a significant decrease and its income per capita fell far below the income of Kenya and Senegal. This difference was accentuated until about 1995. From this year on, Tanzania increased its income per capita, and by 2010 it came very close to the income per capita of Senegal. The income of Senegal did not experience an stable behavior until 1980. That year, it started almost a decade of positive income per capita growth. However, it was followed by a fall in the income per capita of the country that lasted approximately five years. From 1995, it is observed a continuous positive growth in the income per capita of Senegal. In the case of Kenya, the income per capita started below the one of Tanzania, but it had surpassed the latter by 1980. Even though the income per capita of this country did

Figure 3.2: GDP per capita evolution



Source: Own elaboration from PWT

not have a continuous evolution, we can observe that it grew until 1995. In this year, there was a continuous fall that lasted almost a decade. From the early 2000's, this country experienced an unprecedented continuous positive growth.

Table 3.4: Aggregate productivity

	1975	2010
BWA	5482.77	28691.94
CHN	3045.85	16113.85
EGY	4763.28	30675.86
IND	4398.81	11075.12
Mean	4415.93	21639.19
ETH	1483.74	2327.03
MWI	3365.3	2713.35
Mean	2424.52	2520.19
KEN	5532.07	6816.35
SEN	6408.97	6560.69
TZA	4727.37	4643.38
Mean	5556.14	6006.81
USA	61859.73	107893.8

Source: Own elaboration from PWT

Table 3.4 shows the production per worker or productivity for each country and group in 1975 and 2010. Since the number of workers in a country is lower than its total population, figures of aggregate productivity are higher than figures of GDP per capita. We can observe that countries with an initial GDP per capita lower than others, were above the latter in terms of aggregate productivity. This is because of the differences on the labor force of countries. Therefore, countries with lower income per capita and higher aggregate productivity have a lower percentage of working population than countries with higher income per capita and lower aggregate productivity.[1] We can observe that this is the case of Botswana with respect to India, Egypt with respect to Malawi and Kenya and Senegal with respect to Tanzania in 1975.

On average, we can see that the countries with the highest initial aggregate productivity are the ones of the third group. However, there were countries of group 1, Botswana and Egypt, that were more productive than Tanzania. Group 1 was the second most productive, although Malawi was able to surpass the Chinese productivity. In 2010, group 1 has the highest productivity. This group has, on average, a final aggregate productivity almost five times higher the initial one. Within this group, the most remarkable is the evolution of the aggregate productivity in Egypt. On average, groups 2 and 3 also raised their productivity, but at a much lower extent than group 1. Within groups 2 and 3 we can find that Malawi and Tanzania have experienced a decreased in their aggregate productivity. On the contrary, it is noticeable the increase of the aggregate productivity in Ethiopia and Kenya.

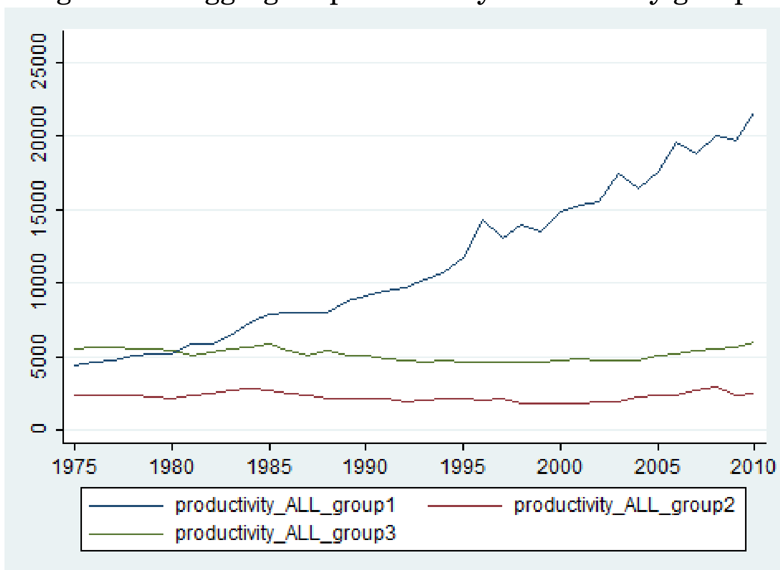
[1] In order to clarify this idea, let us suppose a country i and a country j such that

$$\left(\frac{GDP}{Population}\right)_i < \left(\frac{GDP}{Population}\right)_j$$

$$\left(\frac{GDP}{Workers}\right)_i > \left(\frac{GDP}{Workers}\right)_j .$$

Looking at the aggregate productivity of the USA we can see that countries are still far from it. However, its growth has been surpassed by the growth of the aggregate productivity of the first group.

Figure 3.3: Aggregate productivity evolution by groups



Source: Own elaboration from PWT

Figure 3.3 shows the path followed by the aggregate productivity of each group. Similarly, to what happened in the income per capita of groups, the first group surpassed the third one in the 1980's. Even though it is appreciated a positive trend in the growth of the aggregate productivity of group 1, this has not been as smooth as the evolution of its income per capita. The aggregate productivity of group 3, initially the highest, has maintained around its initial level. The aggregate productivity of group 2, which initially was the lowest, has also remained around its initial level.

Given the similarity between the evolution of the income per capita and the aggregate productivity of countries observed from 1975 to 2010, we will try to analyze whether the behavior of

Alternatively, the GDP per worker can be expressed as

$$\frac{GDP}{Workers} = \frac{GDP}{Population} \cdot \frac{Population}{Workers}.$$

Therefore, it must be fulfilled that

$$\left(\frac{GDP}{Population} \cdot \frac{Population}{Workers}\right)_i > \left(\frac{GDP}{Population} \cdot \frac{Population}{Workers}\right)_j.$$

Since $\left(\frac{GDP}{Population}\right)_i < \left(\frac{GDP}{Population}\right)_j$, to fulfill the previous expresion we need

$$\left(\frac{Population}{Workers}\right)_i > \left(\frac{Population}{Workers}\right)_j.$$

This implies that

$$\left(\frac{Workers}{Population}\right)_i > \left(\frac{Workers}{Population}\right)_j.$$

the sectoral productivity of countries can explain the behavior of their aggregate productivity and income per capita.

Figure 3.4: Aggregate productivity evolution within groups

Source: Own elaboration from PWT

Within groups countries also experienced differences in the paths of their aggregate productivity. In the first group, the aggregate productivity Botswana was able to grow much faster than the other countries (figure 3.4). However, from 1995 this growth has experienced many ups and downs. The growth of the aggregate productivity in Egypt was not so pronounced until the late 1980's, but from that year it has intensified and the aggregate productivity of this country already surpassed the one of Botswana by 2010. The aggregate productivity of China did not start growing significantly until the late 1990's, whereas the aggregate productivity of India did not start until 2005. In group 2, the aggregate productivity of Malawi has maintained over the aggregate productivity of Ethiopia over the whole period analyzed, though the gap has decreased in comparison to 1975. In group 3, it can be appreciated that there was a high gap of Kenya and Senegal with respect to Tanzania until 1995. That year, the aggregate productivity of Tanzania started growing and the country was able to reduce the gap with the other members of group 3.

The evolution of the productivity does not only vary across countries, but is also different across sectors. Table 3.5 presents the average annual growth for the productivity in each sector for every country and group. On average, group 1 is the only group with positive growth rates in every

Table 3.5: Average annual growth rates of the sectoral productivity

	Group 1					Group 2			Group 3				
	BWA	CHN	EGY	IND	**Mean**	ETH	MWI	**Mean**	KEN	SEN	TZA	**Mean**	USA
AGR	2.37	1.97	5.39	1.09	**2.71**	7.28	1.70	**4.49**	0.87	-0.36	0.30	**0.27**	4.74
MIN	8.65	6.11	6.30	2.75	**5.95**	-4.90	2.69	**-1.11**	-14.71	-1.43	0.57	**-5.19**	1.53
MAN	1.37	5.19	5.39	3.25	**3.80**	-1.18	-1.34	**-1.26**	-2.12	-1.59	-2.38	**-2.03**	3.81
PU	5.27	3.16	3.15	4.89	**4.12**	0.89	-0.03	**0.43**	0.53	7.70	-3.03	**1.73**	1.94
CON	4.45	0.71	5.37	-1.57	**2.24**	-3.51	-3.09	**-3.30**	-4.42	0.25	-1.50	**-1.89**	-0.97
WRT	0.63	1.65	4.34	2.02	**2.16**	-0.82	-5.35	**-3.09**	-2	-2.83	-3.11	**-2.65**	2.51
TRA	7.71	3.72	5.97	3.29	**5.17**	3.21	-1.77	**0.72**	0.08	0.18	-2.50	**-0.75**	3.16
FIRE	3.54	5.51	4.12	1.28	**3.61**	1.31	-2.30	**-0.50**	1.78	0.09	-2.68	**-0.27**	0.81
OTH2	-6.03	2.10	6.38	4.17	**1.66**	3.98	-3.98	**0**	-1.41	-0.36	-2.83	**-1.53**	0.40

Source: Own elaboration from GGDC 10-Sector Database

sector. In fact, group 1 has the highest growth rates in the productivity of all sectors, with the exception of agriculture. Mining and transport services are the sectors in which this group has the highest growth rates in productivity. Indeed, except India, the other three countries (Botswana, China and Egypt), have an impressive increase in the productivity in the mining sector. The sector in which they had the lowest rate is personal and government services. This is due to the high decrease of the productivity of Botswana in this sector. Within group 1, we can find that there is a great difference between the productivity growth rates of India and those of the other countries in group 1. India experiences sectoral productivity growth rates much lower than the other countries of its group for all sectors. This may explain why the aggregate productivity and the income per capita of India did not grow like the other members of group 1.
Group 2 has the highest growth in agriculture. Countries of this group have especially decreased their productivity in construction and trade services. It is remarkable the case of Malawi. With the exception of agriculture and mining, this country exhibits a decrease in the productivity of all sectors. Although Ethiopia had a negative evolution in the mining productivity, it achieved high growth rates in utilities and services sectors. Even though Malawi and Egypt had very similar income per capita in 1975, we can see that they have followed very different paths in the sectoral productivity. While Egypt experienced very high growth rates in the productivity of all sectors, Malawi decreased its initial productivity in almost all of them. This difference might explain why Egypt moved to a higher quintile whereas Malawi remained in the lowest one.
With respect to the third group, we can see that, on average, they have only experienced an increase in the productivity of utilities or agriculture. They have especially decreased their productivity in the mining sector (due to the impressive fall in the productivity of the mining sector in Kenya). In this third group, Tanzania is the country that exhibits a productivity fall in a higher number of sectors (all but agriculture and mining). The negative evolution of the sectoral productivity in this group contrasts with the positive one experienced by the first group. Given this difference, it could be explained why the first group surpassed the third one in terms of aggregate productivity and income per capita.
In comparison to the growth rates of the sectoral productivity in USA, we can find that the rates of groups 2 and 3 are below them in all sectors. On the contrary, group 1 surpass the US productivity growth in mining, utilities, construction, transport and finance services.

An important feature that reflects the degree of development in countries is the share of labor allocated to each activity. Usually, poor countries devote most of their resources to produce the basic goods to survive. That is, most of labor is allocated to the agriculture sector. If they are wealthier they can devote part of this labor to manufacturing, reducing the share of labor in agriculture and increasing their productivity. When they become richer, they allocate part of the labor force of manufacturing to the services sectors. Duarte and Restuccia (2010) found in their analysis that countries followed this pattern from 1956 to 2004. Moreover, the authors state that there was a general tendency, independent on the income of countries, that imply a reduction in the share of labor allocated to agriculture and an increase in the share of labor allocated to services.

Table 3.6: Sectoral share of labor

	1975												
	Group 1					Group 2			Group 3				
	BWA	CHN	EGY	IND	**Mean**	ETH	MWI	**Mean**	KEN	SEN	TZA	**Mean**	USA
AGR	1.02	1.16	0.69	1.13	**0.67**	1.03	0.97	**0.88**	1	0.9	1.11	**0.8**	0.03
MIN	2.40	0.8	0	0.8	**0.01**	0	0	**0**	0	0	0	**0**	0.01
MAN	0.23	1.14	1.60	1.03	**0.09**	0.67	1.33	**0.03**	0.82	1.64	0.55	**0.04**	0.2
PU	0	0	4	0	**0**	0	0	**0**	0	0	0	**0**	0.01
CON	1.33	0.67	1.67	0.33	**0.03**	0	2	**0.01**	1	1	1	**0.01**	0.05
WRT	0.63	0.42	1.89	1.05	**0.05**	1	1	**0.03**	0.8	1.6	0.6	**0.05**	0.22
TRA	0.44	0.89	1.78	0.89	**0.02**	0	2	**0.01**	1.2	1.2	0.6	**0.02**	0.06
FIRE	1.33	1.33	1.33	0	**0.01**	0	0	**0**	3	0	0	**0**	0.1
OTH2	1.36	0.38	1.58	0.68	**0.13**	0.86	1.14	**0.04**	1.17	1.3	0.52	**0.08**	0.32

	2010												
	Group 1					Group 2			Group 3				
	BWA	CHN	EGY	IND	**Mean**	ETH	MWI	**Mean**	KEN	SEN	TZA	**Mean**	USA
AGR	0.99	0.96	0.62	1.43	**0.39**	1.07	0.93	**0.7**	0.84	0.89	1.26	**0.57**	0.01
MIN	2	2	0	0	**0.01**	0	0	**0**	1.5	0	1.5	**0.01**	0
MAN	0.5	1.58	0.92	1	**0.12**	1.2	0.8	**0.05**	1.5	1.15	0.35	**0.09**	0.09
PU	2	0	2	0	**0.01**	0	0	**0**	3	0	0	**0**	0
CON	0.4	1.07	1.6	0.93	**0.08**	0.57	1.43	**0.04**	1.13	1.5	0.38	**0.03**	0.05
WRT	1.38	0.73	1.02	0.87	**0.14**	0.87	1.13	**0.12**	1.02	1.34	0.64	**0.16**	0.24
TRA	0.57	0.76	1.71	0.95	**0.05**	0	2	**0.01**	1.13	1.13	0.75	**0.03**	0.04
FIRE	1.74	0.77	0.99	0.5	**0.04**	0	0	**0**	1.5	0	1.5	**0.01**	0.18
OTH2	1.17	1.06	1.39	0.39	**0.18**	0.71	1.29	**0.07**	1.24	0.88	0.88	**0.11**	0.37

Source: Own elaboration from GGDC 10-Sector Database

Table 3.6 shows the average share of labor allocated by groups to each sector. The values for countries represent how much greater or lower they are with respect to the mean of their group. The nearer the values of countries are to 1, the more alike the countries are within a group.[2]

As one might expect from their position in the income per capita distribution in 1975, these countries allocated most part of their labor force to agriculture, especially groups 2 and 3. Personal and government services was the second activity to which all groups allocated more labor. In group

[2]If the countries presented identical characteristics the mean would be the same value as the ones of the countries. Therefore, when dividing their values by the mean, we would obtain 1

1, manufacturing was the third activity that received a higher proportion of labor. Apparently, this group of countries were more industrialized than the countries analyzed in the third group. In 2010, all groups present lower shares of labor allocated to agriculture and larger shares allocated to activities related to services in comparison to the beginning of the period studied, as Duarte and Restuccia (2010) found. It is remarkable the decrease in the share of labor allocated to agriculture by group 1 and the increase in the share of labor allocated to trade services by group 3.

Within group 1, it is noticeable that Egypt presented a distinct allocation of the labor force from the other members of the group in 1975 (closer to a more developed economy). This country allocated a much lower share of labor to agriculture and a higher proportion in manufacturing and all services, but finance services. Botswana also distinguished because of the higher share of labor allocated to mining. Within group 2, there were also differences. Malawi allocated more labor to manufacturing and services than Ethiopia. Nevertheless, both countries allocate a quite similar fraction of their labor force to agriculture. In group 3, it was remarkable the higher share of labor allocated to mining and trade services by Senegal and to finance services by Kenya. As for group 2, all the three countries allocate a similar proportion of their labor force to agriculture. In 2010, Egypt still differentiates from the members of group 1 for the low share of labor allocated to agriculture. This country also distinguishes from the others of its group for the higher shares of labor allocated to some services (construction, transport, personal and government services). Apparently, the only country that do not show up the same transition to a more developed economy is India, with a quite high labor share allocated to the agriculture sector, and a low labor share allocated to services (lower than its group). It is noticeable the behavior of China, which has increased its share of labor allocated to mining until reaching the one of Botswana and has surpassed the share of labor of its group in the manufacturing sector.
In group 2, Ethiopia and Malawi ended up with quite similar labor shares in agriculture and manufacturing but not in services. Kenya differentiates in group 3 because of its higher shares of labor in most services sectors. Senegal distinguishes because of its low shares in mining and finance and its higher shares in construction and trade services. Tanzania differentiates for having higher shares of labor in agriculture and lower shares in most services.

In table 3.7, are presented the sectoral shares of labor of the groups and the ones of the USA. In 1975, we can see that the share of labor allocated to agriculture by the USA was much lower than the shares of the groups. However, the proportion of labor of this country allocated to manufacturing, trade, finance and personal and government services was much higher than the corresponding proportions of groups. In 2010, the initial pattern holds. It is appreciated that by 2010, all groups have achieved a similar proportion of labor allocated to manufacturing than USA and in some services.

Although there has been an increase in the industrial share of the labor force for the groups that equals it to the share of the USA, the agricultural shares are still high, which shows that the countries of this sample have not completed the transition from the agricultural to the industrial sector. Group 1 is the group that has exhibited a higher transition of labor share from agriculture to manufacturing and services (mainly to trade services).

Figure 3.5 illustrates the evolution of the shares of labor allocated by groups to each sector. In the graphs we can see that the share of labor allocated to agriculture has decreased in all groups. It is appreciated that the share of labor in this sector was initially low in group 1 and its gap with the other two groups has widened over the period analyzed. However, group 1 allocated a higher share of labor to the remaining activities, with the exception of trade services, in which group 3 exhibit a higher increase. Graphs show that the activity that experienced the highest increase in

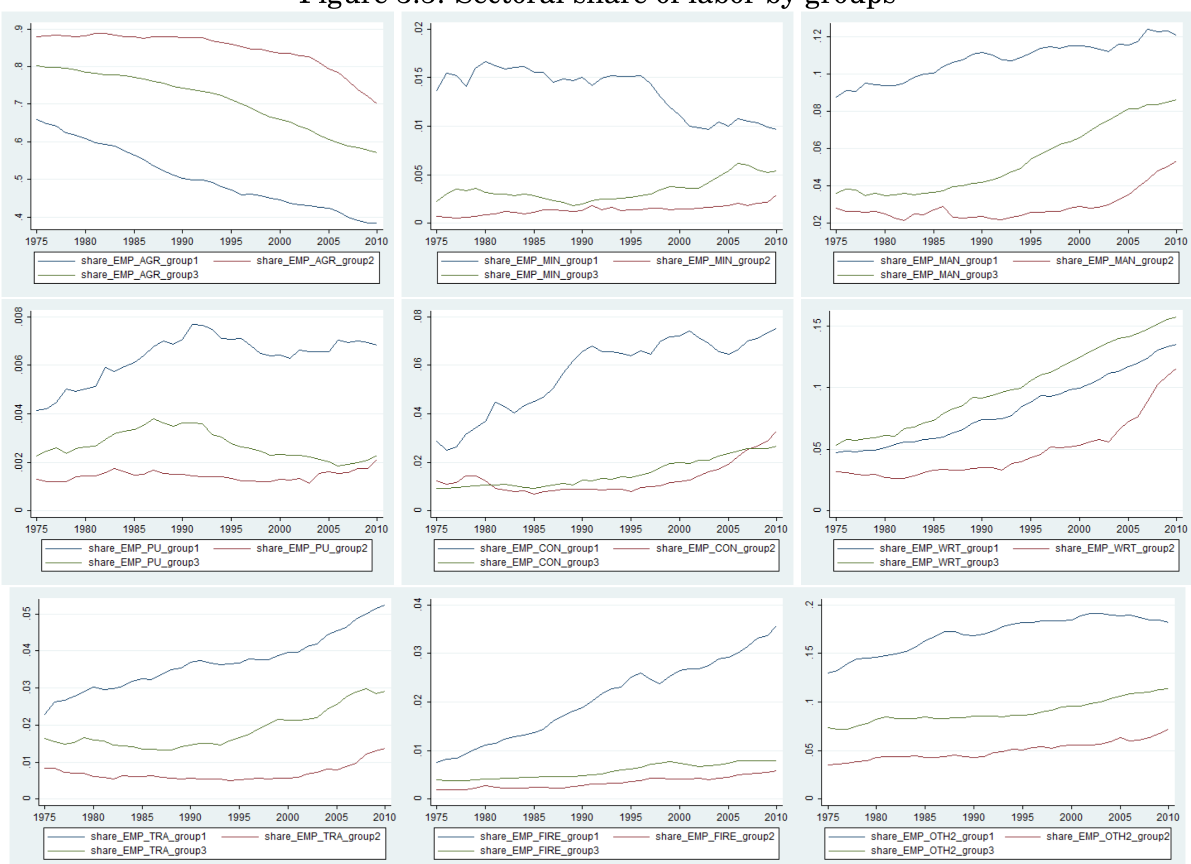
Table 3.7: Comparison of the sectoral shares of labor

1975	Group 1	Group 2	Group 3	USA
AGR	0.67	0.88	0.8	0.03
MIN	0.01	0	0	0.01
MAN	0.09	0.03	0.04	0.2
PU	0	0	0	0.01
CON	0.03	0.01	0.01	0.05
WRT	0.05	0.03	0.05	0.22
TRA	0.02	0.01	0.02	0.06
FIRE	0.01	0	0	0.1
OTH2	0.13	0.04	0.08	0.32

2010	Group 1	Group 2	Group 3	USA
AGR	0.39	0.7	0.57	0.01
MIN	0.01	0	0.01	0
MAN	0.12	0.05	0.09	0.09
PU	0.01	0	0	0
CON	0.08	0.04	0.03	0.05
WRT	0.14	0.12	0.16	0.24
TRA	0.05	0.01	0.03	0.04
FIRE	0.04	0	0.01	0.18
OTH2	0.18	0.07	0.11	0.37

Source: Own elaboration from GGDC 10-Sector Database

Figure 3.5: Sectoral share of labor by groups



Source: Own elaboration from GGDC 10-Sector Database the share of labor allocated to it was trade services.

Chapter 4

The model

In this analysis, we will use a general equilibrium model micro-funded that contains the main ideas of the model proposed by Duarte and Restuccia (2010) and later modifications done by Üngör (2017). Originally, the model was formed by the main 3 sectors (agriculture, industry and services), but Üngör (2017) disaggregated the industry sector into two sectors (mining and manufacturing) and the services sector into six sectors (utilities, construction, trade services, transport services, business services, government and personal services), since this can improve significantly the analysis, as is shown in his article. Therefore, we will commit to the more detailed sectoral decomposition based on 9 different types of activities.

The model is static and its economy is only composed by households or families and firms, there is not a public sector nor an external one. Despite agents make their decisions considering only the present period, we can obtain the dynamics of variables, particularly the dynamics of labor allocation. This is done by assuming that the sectoral productivity growth in the economy is exogenous.

On the one hand, households maximize their utility by deciding each period the amount of consumption of any of the nine type of goods. There are 9 different types of goods and services that households can consume each period. Therefore, there are 9 sectors to which they must provide labor to produce what they consume. The only restrictions that households need to take into account are that their labor income must equal the value of their total consumption each period and that they must consume above a certain level of the agriculture goods to survive. Since there is no capital, households cannot invest in any kind of assets.

On the other hand, firms look for maximizing their profits in perfect competition both, in the labor and in the goods markets.

The economy reaches the equilibrium when all markets, those of goods and services and those of the production factors, clear.

4.1 Description

Firms and technology

The available sectoral technology affects not only the way firms produce goods and services, but also to the amount they produce. In general, it is assumed that the higher the productivity level, the higher the amount of goods and services firms will be able to produce, holding the other factors constant. Therefore, the production function of each sector represents the technological constraints faced by the firms. This model assumes that all the firms in each sector produce with the same technology or production function, which depends on the specific technology of the activity and the share of total employment devoted to it. The following equation reflects the characteristics of such production function for each sector

$$Y_i = A_i \cdot L_i, \text{ for } i = 1, \dots, 8, \text{ agriculture}, \tag{4.1}$$

where Y_i is the production in sector i, A_i its specific technology and L_i the share of total employment in sector i.

The model assumes that firms maximize profits under perfect competition, both in the goods and services and in the labor markets. Consequently and given that firms in each sector produce with the same technology, all firms face the same problem and consider the market prices for goods, services and labor as given. The profits (π) maximization problem of a representative firm of sector for $i = 1, \dots, 8$ and agriculture, can be expressed as

$$\max_{L_i} \left\{ \pi_i = p_i \cdot A_i \cdot L_i - \omega \cdot L_i \right\},$$

where p_i represents the market price for the good or service produced by sector i, ω are the wages received by employees as a payment for their work and L_i is the amount of labor in sector i. Since all firms in each sector behave the same, the decision problem of one of its firms may reflect the behaviour of the whole sector.

Households and preferences

In this model, families or households have an infinite period life. We will normalize the size of households to 1 each period. Every period they try to maximize their utility choosing the amount of goods and services they want to consume. For this purpose, they must take into account the budget constraint, that is, they must consider that their expenditure has to be equal to the income they obtain for providing labor to the firms. Households offer labor to firms inelastically, in other words, they spend all the time they have working because leisure does not give them utility. An important distinctive feature of this model is that it assumes that preferences are non-homothetic when introducing a minimum consumption of agricultural goods needed to survive. Since the only production factor is labor, there is no physical nor human capital, households do not have the possibility to invest in these kind of assets.
Preferences of a representative household can be expressed as

$$\sum_{t=0}^{\infty} \beta^t \cdot u\left(c_t, c_{a,t}\right), \quad \beta \in (0,1)$$

where β represents the discount factor and $u\left(c_t, \bar{a}\right)$ is the utility of households for each period. The discount factor is the relative value attached by households to future levels of consumption and utility in order to express their value in the present. Therefore, greater values of β indicate that households value more the future. The utility depends on the consumption of the agricultural good $c_{a,t}$ and the combination of consumption of the other goods and services c_t as follows:

$$u\left(c_t, c_{a,t}\right) = a \cdot \ln\left(c_t\right) + (1-a) \cdot \ln\left(c_{a,t} - \bar{a}\right).$$

The parameter a indicates what is the relative preference of non-agricultural goods relative to the agricultural ones. The parameter $\bar{a}$ refers to the minimum level of consumption in agriculture needed to survive. The variable $c_{a,t}$ reflects the agricultural consumption while c_t is a combination of the non-agricultural goods and services. This latter combination has the following structure

$$c_t = \left(\gamma_1^{\frac{1}{\eta}} \cdot c_{1,t}^{\frac{\eta-1}{\eta}} + \ldots + \gamma_8^{\frac{1}{\eta}} \cdot c_{8,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}.$$

The problem that households solve every period in order to maximize their utility is

$$\max_{\{c_t, c_{a,t}\}} \left\{ a \cdot \ln \left(\gamma_1^{\frac{1}{\eta}} \cdot c_{1,t}^{\frac{\eta-1}{\eta}} + \ldots + \gamma_8^{\frac{1}{\eta}} \cdot c_{8,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} + (1-a) \cdot \ln(c_{a,t} - \bar{a}) \right\}$$
$$\text{subject to } p_a \cdot c_{a,t} + p_1 \cdot c_{1,t} + \ldots + p_8 \cdot c_{8,t} = \omega$$

where η is the elasticity of substitution between non-agricultural goods or services and γ_i is the weight that the household gives to the consumption of good or service i. Therefore, the sum of γ_i must be equal to one. The budget constraint restriction impose that households cannot save nor have debts, they must consume each period all the labor income they obtain.

Market clearing

The markets in the economy "clear" when their offer and demand coincide. The conditions that must be fulfilled to ensure that markets clear are:

In the labor market

This market will clear if all the labor offered by households equal the labor hired by firms. Therefore, if the firms of every sector i demand a proportion, L_i, of the labor offered by households, the sum of all these proportions must equal 1, which represents the 100% of the labor offered by households.

$$L_a + L_1 + L_2 + \cdots + L_8 = 1 \tag{4.2}$$

In the goods and services market

When the demand of goods and services done by households coincide with the quantity of goods and services produced by firms these markets clear. The following equations reflect those conditions

$$c_a = Y_a,\, c_1 = Y_1,\, c_2 = Y_2,\, \ldots,\, c_8 = Y_8. \tag{4.3}$$

Competitive equilibrium

Given a set of market prices for the goods and services $\{p_a, p_1, p_2, \cdots, p_8\}$ and the wage rate per unit of work $\{\omega\}$ of the economy, the economy will be in equilibrium if the decisions taken by households when maximizing their utility $\{c_a, c_1, c_2, \cdots, c_8\}$ and the ones taken by firms when maximizing profits, $\{L_a, L_1, L_2, \cdots, L_8\}$, are such that all markets clear

$$L_a + L_1 + L_2 + \cdots + L_8 = 1 \text{ and}$$
$$c_a = Y_a,\, c_1 = Y_1,\, c_2 = Y_2,\, \ldots,\, c_8 = Y_8.$$

Model solution

Firms

The solution for the decision problem of the firms is given by its first order condition, which can be obtained by computing the first derivative of the profits with respect to the labor and equating it to 0.

$$\frac{\partial \pi_i}{\partial L_i} = p_i \cdot A_i - \omega = 0 \implies p_i \cdot A_i = \omega.$$

The condition obtained from it shows that firms hire labor until the value of its marginal productivity equals its marginal cost. In other words, firms hire labor until the profit they obtain with the last unit of labor hired $p_i \cdot A_i$ is equal to the cost that generates this last unit of labor to the firm ω. Otherwise, if the revenue they obtained by hiring one more unit of labor is higher (lower) than the cost, the firms would hire one more (less) unit of labor.
If we normalize wages to 1, we can obtain from the first order conditions of firms that the price of goods and services depend inversely on their productivity. In order to fulfill the first order condition and holding wages constant, this implies that the greater (lower) is the available technology in a sector, the lower will be the price (greater) of the good or service produced.

$$p_i = \frac{1}{A_i}, \text{ for } i = 1, \ldots, 8, \text{ agriculture.} \tag{4.4}$$

Households

Initially, we will study how households distribute their non-agricultural consumption among the other goods and services available. Having the optimal combination of consumption for non-agricultural goods and services, we will use it to obtain the optimal agricultural consumption.

Solving for the consumption of non-agricultural goods Firstly, we will analyze how households distribute their consumption among the non-agricultural goods or services such that the cost of the whole non-agricultural consumption is minimized. For this purpose, households minimize the non-agricultural consumption cost given the structure of the non-agricultural consumption. Alternatively, this decision problem can be solved by maximizing the non-agricultural consumption taking into account the budget constraint.

Therefore, we need to solve the following constrained maximization problem:

$$\max_{\{c_i\}_i \geq 0} c_i = \left(\gamma_1^{\frac{1}{\eta}} \cdot c_1^{\frac{\eta-1}{\eta}} + \ldots + \gamma_8^{\frac{1}{\eta}} \cdot c_8^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

$$\text{subject to } p_a \cdot c_a + \sum_{i=1}^{8} p_i \cdot c_i = \omega$$

for $i = 1, \ldots, 8$.

Normalizing wages to 1, it would be expressed as

$$\max_{\{c_i\}_i \geq 0} c_i = \left(\gamma_1^{\frac{1}{\eta}} \cdot c_1^{\frac{\eta-1}{\eta}} + \ldots + \gamma_8^{\frac{1}{\eta}} \cdot c_8^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

$$\text{subject to } p_a \cdot c_a + \sum_{i=1}^{8} p_i \cdot c_i = 1$$

To solve this problem we will use the Lagrange multiplier

$$\mathcal{L} = \left(\sum_{i=1}^{8} \gamma_i^{\frac{1}{\eta}} \cdot c_i^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} + \lambda \cdot \left(1 - p_a \cdot c_a - \sum_{i=1}^{8} p_i \cdot c_i \right).$$

and differentiate it with respect to the different types of non-agricultural consumption

$$\frac{\partial \mathcal{L}}{\partial c_i} = \frac{\eta}{\eta - 1} \cdot \left(\sum_{i=1}^{8} \gamma_i^{\frac{1}{\eta}} \cdot c_i^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}-1} \cdot \gamma_i^{\frac{1}{\eta}} \cdot c_i^{\frac{\eta-1}{\eta}-1} \cdot \frac{\eta - 1}{\eta} - \lambda \cdot p_i = 0, \quad \forall i = 1, \ldots, 8$$

Through simple algebraic manipulation (see Appendix A), we obtain the following expression for the non-agricultural relative consumption.

$$\frac{c_j}{c_i} = \left(\frac{p_i}{p_j} \right)^{\eta} \cdot \frac{\gamma_j}{\gamma_i} \tag{4.5}$$

Rearranging terms of the first order conditions we can see that household will decide how much to consume of each good or service depending not only on its preferences $\frac{\gamma_j}{\gamma_i}$, but also on its relative price $\frac{p_i}{p_j}$. The greater are the preferences for one good or service, the higher will be the consumption of that good or service. As one might expect also, the greater the price of a good or service with respect to the price of another, the lower will be the consumption of the first with respect to the consumption of the second good or service.

By considering the clearing condition for the labor market (equation 4.2), the production function (equation 4.1), the clearing condition for the goods market (equation 4.3), the equation for the relative non-agricultural consumption (equation 4.5) and the equation from the firm's decision problem (equation 4.4) we get the following expression for the non-agricultural shares of employment (see Appendix A).

$$L_i = \frac{\gamma_i A_i^{\eta-1} (1 - L_a)}{\sum\limits_{i=1}^{8} \gamma_i A_i^{\eta-1}}, \tag{4.6}$$

which can be expressed as

$$L_i = \frac{1 - L_a}{1 + \sum\limits_{j=1,\, j \neq i}^{8} \left(\frac{A_i}{A_j} \right)^{1-\eta} \frac{\gamma_j}{\gamma_i}}.$$

This alternative way of expressing the sectoral labor allocation shows that our expressions are consistent with those of Duarte and Restuccia (2010). Implementing the clearing condition for the goods and services market (equation 4.3) and the production function (equation 4.1), we can obtain an expression for the total consumption of non-agricultural goods and services in terms of the sectoral employment.

$$C = \left(\sum_{i=1}^{8} \gamma_i^{\frac{1}{\eta}} \cdot c_i^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \Longrightarrow$$

Implementing the clearing conditions for the goods and services markets

$$C = \left(\sum_{i=1}^{8} \gamma_i^{\frac{1}{\eta}} \cdot Y_i^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \Longrightarrow$$

Implementing the production function

$$C = \left(\sum_{i=1}^{8} \gamma_i^{\frac{1}{\eta}} \cdot (A_i \cdot L_i)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} .$$

Substituting the expressions for the sectoral employment obtained previously (equation 4.6) we can express the total consumption of non-agricultural goods in terms of the labor force in the agriculture sector, which will help us to determine the consumption of agricultural goods (see Appendix A).

$$C = \left(\frac{\sum_{i=1}^{8} \gamma_i A_i^{\eta-1}}{\left(\sum_{i=1}^{8} \gamma_i A_i^{\eta-1} \right)^{\frac{\eta-1}{\eta}}} (1 - L_a)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

Applying logarithms,

$$\ln C = \left(\frac{\eta}{\eta - 1} \ln \left(\frac{\sum_{i=1}^{7} \gamma_i A_i^{\eta-1}}{\left(\sum_{i=1}^{8} \gamma_i A_i^{\eta-1} \right)^{\frac{\eta-1}{\eta}}} \right) + \ln (1 - L_a) \right) . \tag{4.7}$$

Solving for the consumption of agricultural good Secondly, we can solve for the optimal decision between the combination of all non-agricultural goods and the consumption of the agricultural good, taking into account that we have obtained, in the first step, the optimal expenditure among all non-agricultural goods for a given combination of all non-agricultural goods. For this purpose, we will substitute the expression for the logarithm of the combination of non-agricultural consumption (equation 4.7) in the household's decision problem[1] (see Appendix A).

$$\max_{L_a \geq 0} U = \max_{L_a \geq 0} (1 - a) \cdot \ln (c_a - \bar{a}) + a \cdot \ln \left[\gamma_1^{\frac{1}{\eta}} \cdot c_1^{\frac{\eta-1}{\eta}} + \ldots + \gamma_8^{\frac{1}{\eta}} \cdot c_8^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \Longrightarrow$$

$$\max_{L_a \geq 0} U = \max_{L_a \geq 0} (1 - a) \cdot \ln (A_a L_a - \bar{a}) + a \cdot \frac{\eta}{\eta - 1} \ln \left(\frac{\sum_{i=1}^{7} \gamma_i A_i^{\eta-1}}{\left(\sum_{i=1}^{8} \gamma_i A_i^{\eta-1} \right)^{\frac{\eta-1}{\eta}}} \right) + \ln (1 - L_a) .$$

[1]Given that this model assumes that there are no distorsions, the solution obtained from the equilibrium and the optimal solution coincide. Therefore, the equilibrium conditions can be introduced in the household's decision problem.

Now, we can solve for the optimal decision by differentiating with respect to the labor in agriculture, we can obtain an expression for it.

$$\begin{aligned}
\frac{\partial U}{\partial L_a} = \frac{(1-a)\cdot A_a}{A_a \cdot L_a - \bar{a}} - a \cdot \frac{1}{1-L_a} = 0 \Longrightarrow \\
\frac{(1-a)\cdot A_a}{A_a \cdot L_a - \bar{a}} = a \cdot \frac{1}{1-L_a} \Longrightarrow \\
(1-L_a)\cdot(1-a)\cdot A_a = a\cdot(A_a \cdot L_a - \bar{a}) \Longrightarrow \\
(1-a) = a \cdot L_a + (1-a)\cdot L_a - \frac{a\cdot\bar{a}}{A_a} \Longrightarrow \\
L_a = (1-a) + \frac{a\cdot\bar{a}}{A_a}. \qquad (4.8)
\end{aligned}$$

The expressions for the sectoral employment obtained in this section are similar to the ones of Duarte and Restuccia (2010) and Üngör (2017). In the Appendix B, we check if the labor market clears by introducing the expressions obtained for the sectoral employment.

4.2 Calibration

The parameters of the model can be classify in two main groups. On the one hand, we can find the values of the parameters that determine households' preferences (a, γ and η). On the other hand, we can find the values of the technological parameters, particularly those reflecting the initial sectoral productivity (A_i). Following Duarte and Restuccia (2010) and Üngör (2017), we will obtain these parameters using as a reference the USA data for 1975.

Once we have calibrated these parameters, they will allow us to simulate the productivity, income per capita and sectoral labor allocation paths of the countries for the period $1975-2010$.

Calibration of the preferences parameters

The parameters that determine the preferences of households are considered to be the same across countries and constant over time. Otherwise, it would be difficult to determine how much of the changes in the sectoral labor allocation were caused by fluctuations in the sectoral productivity or in the preferences.

Calibration of a

The parameter that weights the utility between agricultural and non-agricultural consumption, $1-a$, will be calibrated to mimic the share of labor allocated to agriculture in the USA in the long run. Following Duarte and Restuccia (2010), $1-a$ will be equal to 0.01. Consequently, a will be 0.99.

Calibration of γ

With the aim of simplifying the analysis, we will normalize the initial productivity across sectors for the USA, setting their initial parameters equal to 1, $A_{i,1975} = 1$ for $i = a, 1, \dots, 8$. Given this normalization, we will consider the equation for the non-agricultural shares of labor (equation 4.6) to obtain an expression for the weights of the utility among the non-agricultural goods and services consumption, γ_i. Considering that these weights add up to 1, these equations show that γ_i can be calibrated as the share of labor allocated to the corresponding sector i without considering the agriculture observed for the USA in 1975 .

$$L_i = \frac{\gamma_i A_i^{\eta-1}\left(1-L_a\right)}{\sum\limits_{i=1}^{8}\gamma_i A_i^{\eta-1}} \Longrightarrow$$
$$L_i = \frac{\gamma_i \cdot \left(1-L_a\right)}{\sum_{i=1,\ldots,8}\gamma_i} \Longrightarrow$$
$$L_i = \gamma_i \cdot \left(1-L_a\right) \Longrightarrow$$
$$\gamma_i = \frac{L_i}{1-L_a} \qquad \forall i \in [1,\, 8]$$

Table 4.1 presents the values of these parameters.

Table 4.1: Calibration of parameters

Parameter	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	γ_8
Value	0.0086	0.2045	0.0069	0.0538	0.2291	0.0593	0.1057	0.3322

Calibration of $\bar{a}$

Introducing the value calibrated for a in the equation of labor in agriculture (equation 4.8), we can obtain the value of the parameter $\bar{a}$ as follows

$$\bar{a} = \frac{L_a - (1-a)}{a} \Longrightarrow$$
$$\bar{a} = \frac{(2871.24/92032.1) - 0.01\,(1+0)}{0.99} \Longrightarrow$$
$$\bar{a} = 0.0214.$$

Calibration of η

To estimate the value for η we will impose that the estimated average annual growth for the US aggregate productivity is equal to the observed one. To calculate the average annual growth of aggrergate productivity, we will introduce the previous parameters and the required data for the USA in 1975 in the expressions for the aggregate productivity. The condition considered to obtain the value of η is

$$\left(\frac{A_{2010}\,(\eta)}{A_{1975}\,(\eta)}\right)^{\frac{1}{35}} - 1 = 1.016 \Longrightarrow$$
$$\left(\frac{A_{2010}\,(\eta)}{A_{1975}\,(\eta)}\right)^{\frac{1}{35}} - 0.016 = 0,$$

where the aggregate productivity of year t is given by the sectoral productivity of the USA in period t, $t = 1975, 2010$, and the sectoral shares of labor estimated for the period t as functions of

η, given this data.

$$A_t(\eta) = \sum_{i=a,1,\ldots,8} A_{i,t} \cdot L_{i,t}(\eta)$$

The sectoral shares of labor in terms of η are given by the expressions of the sectoral employment obtained in the model.[2]

$$L_{a,t} = (1-a) + \frac{a \cdot \bar{a}}{A_{a,t}}$$

$$L_{i,t} = \frac{\gamma_i A_{i,t}^{\eta-1}(1-L_{a,t})}{\sum\limits_{i=1}^{8} \gamma_i A_{i,t}^{\eta-1}} \qquad i = 1, \ldots, 8$$

Following the process described, we obtain that η is equal to 0.2301.

Calibration of the initial sectoral productivity

Having calibrated the values for the preferences parameters of the model and considering the initial sectoral shares of labor and aggregate productivity observed for each country, we can obtain the values of the initial sectoral productivity for each country through the following system of equations

$$L_{i,1975} = \frac{\gamma_i A_{i,1975}^{\eta-1}(1-L_{a,1975})}{\sum\limits_{i=1}^{8} \gamma_i A_{i,1975}^{\eta-1}} \qquad i = 1, \ldots, 8$$

$$L_{a,1975} = (1-a) + \frac{a \cdot \bar{a}}{A_{a,1975}}$$

$$A_{1975} = \sum_{i=a,1,\ldots,8} A_{i,1975} \cdot L_{i,1975}$$

where i refers to the sectors i. Given the lack of information on the people engaged (number of workers) for Botswana from 1975 to 1979, we will measure the aggregate productivity of this country with the GGDC 10-Sector database as the total value added divided by the total employment.

Following Duarte and Restuccia (2010) and Üngör (2017), aggregate productivity will be measured in relative terms to the USA. As a consequence, the resulting sectoral productivity will be measured in terms of the US productivity. Measuring these variables in relative terms to the USA will allow us to minimize the inconvenient caused by the lack of sectoral information in PPP terms.

Table 4.2 shows our estimations for the initial level of labor productivity in relative terms to the USA. According to these estimations, countries had already a great productivity in finance, whereas it was very low in agriculture. With the exception of Botswana, our estimations reflect that countries of group 2 had higher sectoral productivity than countries of group 1. Countries of group 3 had the highest productivity, although Botswana surpassed them in some activities.

[2] In this case, the initial sectoral productivity will not be normalized, it will take its corresponding value. However, it would be similar to normalize it and use the corresponding final value. This is because the ratio between them is used. Since in both cases, the growth rate is the same, the ratio between the final and the initial value would be the same.

Table 4.2: Estimated sectoral productivity levels in 1975

	AGR	MIN	MAN	PU	CON	TRA	WRT	FIRE	OTH2
BWA	0.0318	0.0311	4.4799	0.3173	0.2939	2.9204	1.3027	4.1324	0.4007
CHN	0.0278	0.0084	0.0395	0.0875	0.0768	0.2990	0.0891	0.5827	0.2077
EGY	0.0470	0.2476	0.0705	0.0406	0.0474	0.1509	0.0781	0.9487	0.0802
IND	0.0236	0.1383	0.0289	0.0316	0.0533	0.0145	0.0404	0.4487	0.0231
ETH	0.0296	0.0714	0.0909	0.0959	0.2286	0.2242	0.0969	2.1054	0.1843
MWI	0.0269	0.2233	0.4198	0.2442	0.4840	0.3424	0.2051	1.1812	0.2384
KEN	0.0252	0.2167	0.1490	0.0814	0.0556	0.2362	0.1263	2.0248	0.2871
SEN	0.0299	0.4005	0.2623	0.0972	0.3471	0.1944	0.1734	9.0918	0.2459
TZA	0.0240	0.1261	0.7088	0.3710	0.3349	0.3293	0.3211	3.9125	0.4762

Computation of the sectoral productivity path

The initial values of the sectoral productivity will allow us to to obtain the whole path of the sectoral productivity. We can obtain the value for the sectoral productivity of next years by multiplying the initial values with the growth in the corresponding sectoral productivity. In order to obtain the growth rate in the corresponding sectoral productivity, we calculate the sectoral productivity as the corresponding sectoral value added (VA) divided by the employment of that sector (EMP). [3] The sectoral productivity growth rate for a period t, is calculated as

$$\frac{\frac{VA_{i,t}}{EMP_{i,t}}}{\frac{VA_{i,t-1}}{EMP_{i,t-1}}} - 1 \qquad t = 1976, \ldots, 2010.$$

Table 4.3 shows the average of the annual growth rates of the sectoral productivity for each country. The figures in this Table differ from those in Table 3.5 because the sectoral productivities in this Table are not PPP adjusted. Nevertheless, again, it is noticeable that the only countries that experienced a positive growth in all the sectors are the ones of group 1.

Table 4.3: Average annual growth of sectoral productivity(%)

	AGR	MIN	MAN	PU	CON	TRA	WRT	FIRE	OTH2
BWA	2.96	6.69	9.57	5.29	5.24	2.83	6.98	5.81	3.24
CHN	1.75	7.03	6.12	4.87	1.72	2.16	3.56	5.25	2.34
EGY	2.89	6.33	3.00	3.66	3.33	2.17	4.16	2.87	3.80
IND	1.47	3.32	3.52	4.73	0.18	2.35	3.19	1.11	4.62
ETH	0.01	-5.33	-0.87	-0.07	-3.04	-1.93	1.90	3.40	1.65
MWI	1.42	5.82	0.26	2.68	-0.56	-2.96	0.42	1.57	-1.28
KEN	0.16	-1.82	-1.38	0.61	-3.57	-1.67	-0.14	1.60	-1.75
SEN	-0.25	-1.52	-2.21	6.04	-0.44	-3.70	-0.81	0.45	-1.17
TZA	1.07	-1.74	-0.59	0.01	-2.28	-3.07	-0.25	-0.18	0.71
USA	4.38	1.30	3.04	2.36	-1.77	1.86	2.71	0.44	-0.08

[3] In contrast to the sectoral productivity of the initial analysis, in this case the sectoral productivity is not adjusted in PPP terms. If we obtained the growth rate from the data measured in PPP, it would not only reflect the changes in the sectoral production but it also would show the changes in the international relative prices.

4.3 Simulation

Given the values we have obtained for the parameters and the observed data, we can simulate the evolution of the sectoral shares employment, the relative aggregate productivity and income per capita of countries from 1975 to 2010. By comparing these series with the real data, we will get the accuracy of the model. Later on, comparing these simulated series with their corresponding paths when changing the sectoral productivity, we will observe the relevance of each sector in the evolution of countries.

4.3.1 Simulation of the sectoral shares of employment

Having calculated the sectoral productivity for all years, we can obtain the simulated series for the sectoral shares of employment for $1975-2010$ by substituting the corresponding values in the expressions for the sectoral shares of labor obtained in the model

$$L_{a,t} = (1-a) + \frac{a \cdot \bar{a}}{A_{a,t}} \qquad t = 1976, \ldots, 2010$$

$$L_{i,t} = \frac{\gamma_i A_{i,t}^{\eta-1} (1 - L_{a,t})}{\sum_{i=1}^{8} \gamma_i A_{i,t}^{\eta-1}} \qquad i = 1, \ldots, 8;\ t = 1976, \ldots, 2010.$$

By comparing the resultant shares of labor with the observed ones, we will have an idea on the performance of the model. This is, whether the model gives or not simulations close to the data.

4.3.2 Simulation of the relative aggregate productivity and the income per capita of countries

Once we have the exogenous series for the sectoral productivity and the simulated series for the shares of labor, we can obtain the relative aggregate productivity of countries. The following expression gives the relative aggregate productivity for the country j and the period t

$$Y_{j,t} = A_{aj,t} \cdot L_{aj,t} + A_{1j,t} \cdot L_{1j,t} + \cdots + A_{8j,t} \cdot L_{8j,t}. \tag{4.9}$$

The relative aggregate productivity of countries can be easily obtained by plugging the corresponding simulations for the sectoral shares of labor (L) and the computed sectoral productivity into the expression 4.9. Therefore, a sectoral productivity path will imply a specific evolution of the aggregate productivity of a country. By comparing the changes in the relative aggregate productivity resultant from different sectoral productivity paths, we can identify the sector responsible for the good performance of countries. In addition, we can observe what have been the sectors leading the evolution of the relative aggregate productivity of countries.

The income per capita of countries can be estimated as follows

$$GDPpc_{j,t} = Y_{j,t} \cdot y_{USA,t} \cdot \frac{L_{j,t}}{Pop_{j,t}}$$

[4]where $GDPpc_{j,t}$ refers to the income per capita of country j in period t, $Y_{j,t}$ is the relative aggregate productivity of country j in period t, $y_{USA,t}$ is the aggregate productivity of the USA in the year t, $L_{j,t}$ are the workers of country j in period t and Pop is the population of country j in period t.

Changes in the income per capita of countries also allow us to see which sectors are responsible for their good performance, in terms of income per capita.

4.4 Evaluation of the model

The results obtained through the model can be close or really far from what has really happened. Following Duarte and Restuccia (2010) and Üngör (2017), we will evaluate the model in two different ways.

To know if model is able to replicate what has happened in the countries, firstly, we will look at the figures 4.1 and 4.2. The better performs the model, the more alike will be the data and the simulations. If the simulation and the data coincide, the country will be represented on the 45-degree line of the graphs. If countries are above that line, it indicates that the simulations of the variable are lower than the corresponding values observed in the data. On the contrary, if countries are below the line, the simulations are higher than the data.

In figure 4.1 is presented the similarity between the data and the simulations of the sectoral shares of labor for 2010. In general, the simulations are not very accurate. However, the model gives very precise simulations for the sectors of utilities, mining, except for China, for finance except BWA and USA, and it gives quite good simulations for personal and government services. The model also performs very accurately in specific cases like the USA in agriculture, Botswana in manufacturing and China, Malawi, Tanzania, Ethiopia and Senegal in the finance sector. In addition, this figure contains a comparison between the data and the simulations of the final aggregate productivity. In this case, the model is able to replicate the data better than in most sectors.

Figure 4.2 shows how is the percentage change observed in the sectoral shares of labor from 1975 to 2010 and what is the predicted percentage change in each case. In general, we can see that the simulated changes in labor allocation are not the same as the observed ones, but in some cases are quite close. In mining, simulations are very alike to data for almost all countries but Ethiopia. In construction, simulations are good except for Ethiopia and Senegal. In general, simulations for personal and government services are quite close to the observed data. However, the simulations for the changes in the sectoral shares of labor replicate better the data than the simulations for their levels in 2010. Paying attention to the performance of the model by countries,

[4]Given the expressions for the income per capita and the aggregate productivity we have that

$$GDPpc_{i,t} = Y_{i,t} \cdot y_{USA,t} \cdot \frac{L_{i,t}}{Pop_{i,t}}$$

$$GDPpc_{i,t} = \frac{\frac{GDP_{i,t}}{L_{i,t}}}{\frac{GDP_{USA,t}}{L_{USA,t}}} \cdot \frac{GDP_{USA,t}}{L_{USA,t}} \cdot \frac{L_{i,t}}{Pop_{i,t}}$$

$$GDPpc_{i,t} = \frac{GDP_{i,t}}{L_{i,t}} \cdot \frac{L_{i,t}}{Pop_{i,t}}$$

$$GDPpc_{i,t} = \frac{GDP_{i,t}}{Pop_{i,t}}.$$

Figure 4.1: Comparison between the simulated and the observed sectoral shares of labor

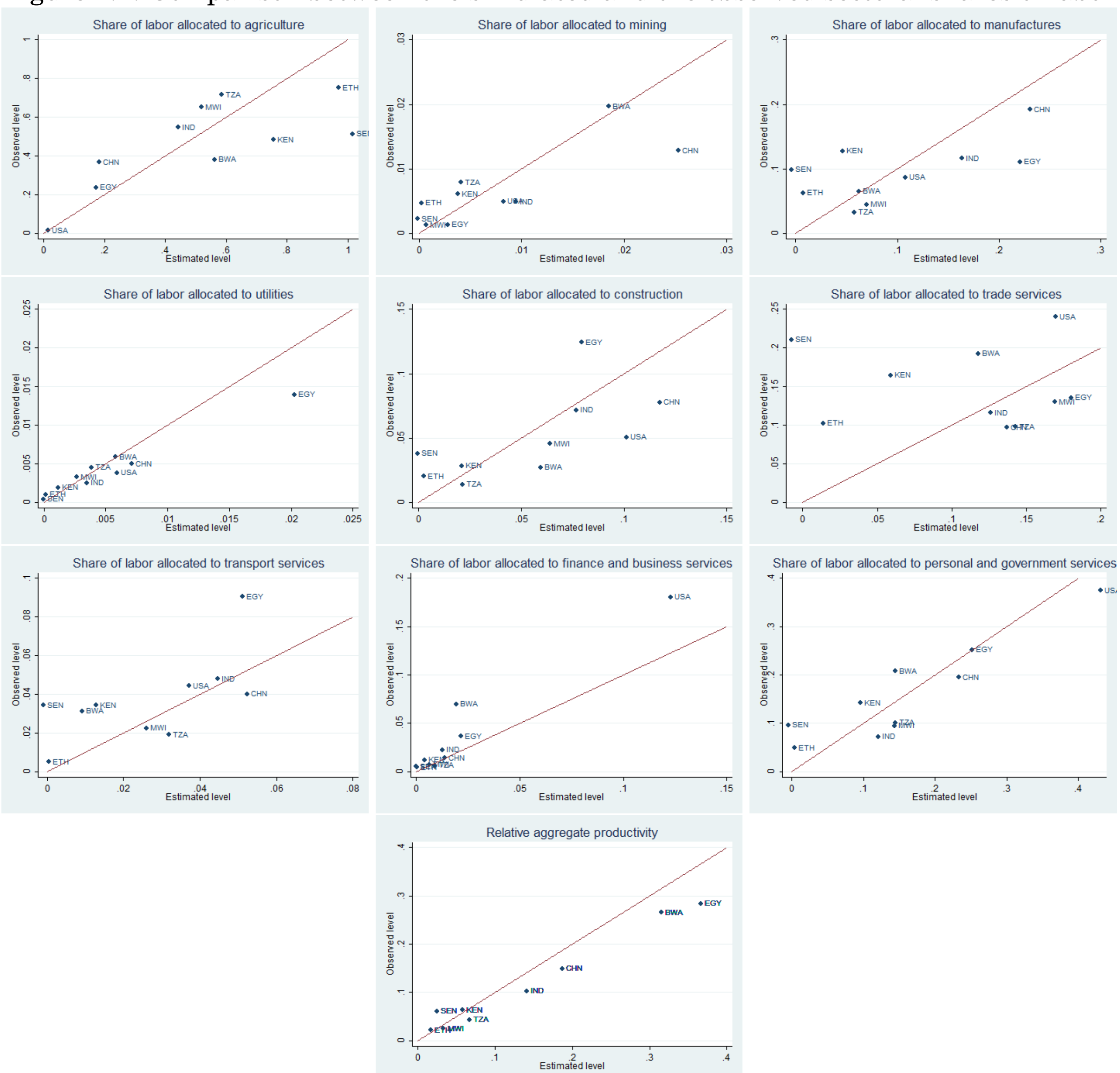
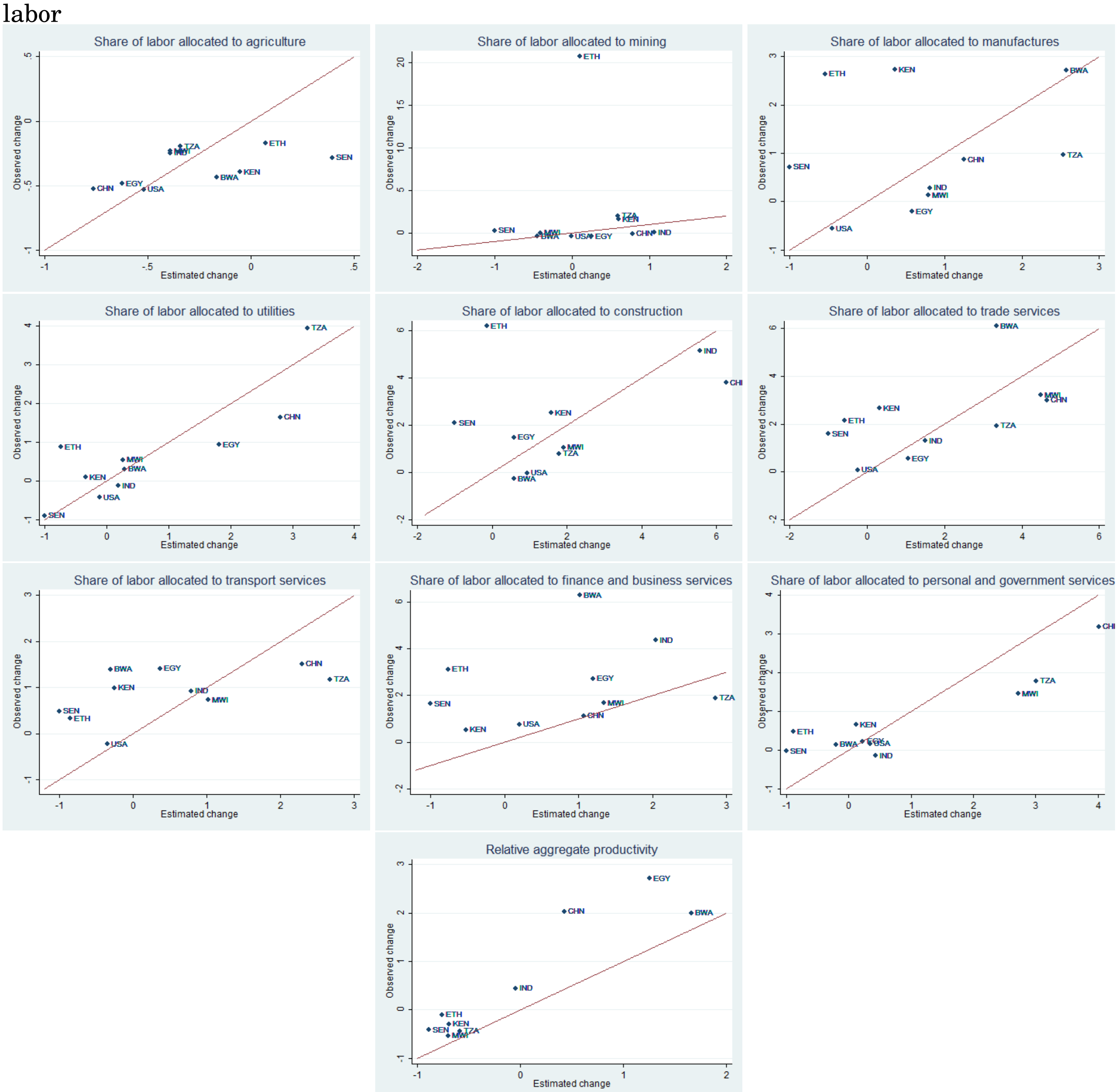


Figure 4.2: Comparison between the simulated and the observed changes in sectoral shares of labor



we can find that it performs very accurately for the USA. Though at a lower extent than the USA, the model also gives very accurate simulations for India and Malawi. On the contrary, simulations for Ethiopia are very distinct from data. This figure, contains also a comparison of the observed and simulated change in the aggregate productivity relative to the USA. In this aspect, the model performs better in simulating the levels for 2010 than the changes of this variable over the period analyzed.

Secondly, we present several statistics in table 4.4 that show more formally the performance of the model.

In the first row of each country is presented the sectoral Root Mean Square Error (RMSE) statistic. The RMSE is computed as

$$\sqrt{\frac{\sum_{t=1}^{T}\left(z_t-\hat{z}_t\right)^2}{T}},$$

where T is the number of years in the period considered, z_t is the observed sectoral share of labor in period t and $\hat{z}_t$ is the simulated sectoral share of labor for the same period. Since the difference between the observed and the simulated value is elevated to the second power, the statistic will always be positive. When the difference between the data and the simulation from the model is large, the statistic takes values far from 0. Therefore, when the statistic is close to 0, the difference between the data and the model is lower, this is, the simulations of the model are more accurate.

In table 4.4, we can find that, in general, the values for this statistic are low. Mining and utilities sectors are the activities that present the best results, while agriculture and personal and government services are the sectors with higher figures. With respect to the relative aggregate productivity, this statistic takes low values. In general, this statistic shows that the countries for which the model gives more accurate simulations are the USA and India.

The second row of each country contains the sectoral Modified Nash-Sutcliffe Efficiency criterion (mNSE). This statistic derivates from the Nash-Sutcliffe efficiency criterion (NSE), which is calculated as

$$1-\frac{\sum_{t=1}^{T}\left(z_t-\hat{z}_t\right)^2}{\sum_{t=1}^{T}\left(z_t-\mu_z\right)^2}.$$

It measures the distance between the data and the simulations with respect to the distance between the data and the mean. When simulations and data coincide, the statistic takes value 1. The greater is the distance between the simulation and the data, the lower is the value. However, this statistic is affected by the scale of the data. Given the consideration of values to the second power, differences in data with high values can be overestimated, while differences in data with low values can be considered 0. The mNSE criterion avoids this inconvenient by considering the differences in absolute value instead of using the square.

$$mNSE=1-\frac{\sum_{t=1}^{T}\left(|z_t-\hat{z}_t|\right)}{\sum_{t=1}^{T}\left(|z_t-\mu_z|\right)}$$

We can see that the values that this statistic takes with the data analyzed are far from 1. It is remarkable the figures of this statistic for India, country for which the statistic takes positive values in most sectors. In general, this statistic indicates that simulations for the sectoral shares of labor and relative aggregate productivity are not very good.

Table 4.4: Statistics to evaluate the performance of the model

	AGR	MIN	MAN	PU	CON	TRA	WRT	FIRE	OTH2	Rel. Aggr. Productivity
Botswana										
RMSE	0.32	0.02	0.02	0.01	0.03	0.06	0.02	0.03	0.16	0.13
mNSE	-2.01	-2.79	-0.06	-1.04	0.20	-0.05	-1.46	-0.92	-8.88	-0.93
md	0.25	0.22	0.51	0.33	0.65	0.50	0.28	0.33	0.09	0.33
CORR	0.21	0.19	0.71	0.23	0.71	0.68	-0.09	0.17	-0.61	0.26
China										
RMSE	0.12	0.01	0.03	0	0.03	0.03	0.01	0	0.02	0.05
mNSE	-0.18	-2.76	-1.25	-0.45	-0.44	-0.03	-0.24	0.24	0.50	-1.11
md	0.59	0.19	0.44	0.53	0.55	0.65	0.59	0.70	0.78	0.32
CORR	0.99	0.18	0.88	0.93	0.98	0.98	0.96	0.87	0.99	0.68
Egypt										
RMSE	0.06	0	0.06	0	0.02	0.04	0.02	0.01	0.01	0.06
mNSE	0.09	-0.33	-9.73	-1.17	-0.34	-0.88	-1.74	-0.20	0.44	0.06
md	0.66	0.27	0.08	0.40	0.40	0.40	0.27	0.06	0.65	0.57
CORR	0.98	0.03	-0.48	0.91	0.72	0.91	0.52	0.71	0.80	0.97
Ethiopia										
RMSE	0.42	0	0.09	0	0.02	0.17	0.01	0	0.12	0.03
mNSE	-10.34	-2.15	-5.64	-16.33	-3.33	-10.36	-30.22	-3.49	-23.72	-3.71
md	0.08	0.24	0.13	0.05	0.19	0.08	0.03	0.45	0.04	0.27
CORR	-0.06	-0.46	-0.16	-0.41	-0.09	0.05	0.26	0.04	-0.47	0.70
India										
RMSE	0.06	0	0.02	0	0	0.01	0.01	0.01	0.02	0.03
mNSE	0.20	-1.01	-1.10	-2.50	0.88	0.60	0.29	0.39	-2.73	-1.85
md	0.71	0.41	0.48	0.14	0.94	0.82	0.68	0.18	0.09	0.26
CORR	0.96	0.47	0.91	-0.46	0.99	0.97	0.94	0.97	-0.43	0.78
Kenya										
RMSE	0.20	0	0.05	0	0	0.06	0.01	0.01	0.05	0.05
mNSE	-0.69	0.31	-0.42	-1.67	0.36	-0.62	-0.58	-1.61	-1.58	-1.17
md	0.37	0.62	0.41	0.27	0.67	0.38	0.39	0.64	0.28	0.44
CORR	0.54	0.92	0.79	0.14	0.86	0.65	0.49	-0.61	0.17	0.88
Malawi										
RMSE	0.18	0	0.04	0	0.02	0.05	0.01	0.01	0.05	0.04
mNSE	-2.43	-1.88	-5.50	-4.53	-2.27	-0.80	-2.10	-1.18	-2.27	-2.90
md	0.35	0.08	0.15	0.09	0.35	0.50	0.36	0.38	0.34	0.20
CORR	0.70	-0.44	0.25	-0.18	0.71	0.78	0.61	0.45	0.69	0.56
Senegal										
RMSE	0.54	0	0.10	0.01	0.03	0.22	0.03	0	0.15	0.06
mNSE	-7.18	-4.18	-5.78	-1.52	-2.24	-4.20	-6.15	-2.76	-37.83	-2.48
md	0.10	0.20	0.13	0.29	0.23	0.16	0.11	0.20	0.02	0.31
CORR	-0.40	0.44	-0.25	-0.30	-0.27	-0.52	-0.03	-0.36	0.17	0.77
Tanzania										
RMSE	0.09	0	0.02	0	0.01	0.03	0.01	0	0.03	0.04
mNSE	-1.06	-0.70	-2.25	0.66	-1.44	-1.01	-1.74	-0.10	-0.85	-1.42
md	0.46	0.34	0.32	0.85	0.41	0.49	0.35	0.59	0.49	0.29
CORR	0.90	0.16	0.83	0.97	0.85	0.95	0.83	0.90	0.88	0.90

USA									
RMSE	0	0	0	0	0.02	0.05	0.01	0.04	0.06
mNSE	0.63	-0.42	0.75	-0.34	-3.93	-9.06	-0.44	-0.32	-2.62
md	0.82	0.41	0.86	0.43	0.19	0.09	0.51	0.43	0.22
CORR	0.97	0.79	0.99	0.87	0.41	-0.45	0.90	0.92	0.85

In the third row we can find the modified index of agreement (md), which is based on the index of agreement (d).

$$1 - \frac{\sum_{t=1}^{T} (z_t - \hat{z}_t)^2}{\sum_{t=1}^{T} (|\hat{z}_t - \mu_z| + |z_t - \mu_z|)^2}$$

When this statistic takes value 1, it implies that the data and the simulations coincide. As a result, the difference between them is 0, which reflect that the model can replicate the data perfectly. When this statistic is close to 0, it reflects that the simulations of the model are far from the data. Similarly to what happened in the previous statistic, it is affected by the scale of the data. Due to the squares, differences of large values will be intensified, while differences of low values will be reduced. The md statistic substitutes the square of the differences by their absolute value in order to solve this problem.

$$md = 1 - \frac{\sum_{t=1}^{T} (|z_t - \hat{z}_t|)}{\sum_{t=1}^{T} (|\hat{z}_t - \mu_z| + |z_t - \mu_z|)}$$

This statistic shows that the model successes in replicating the sectoral shares of labor for USA, Tanzania and India. In terms of the relative aggregate productivity, this statistic reflects that the performance of the model is not so good.

The last row of the countries show the coefficient of correlation between countries. This statistic measures the linear relationship of two variables. In this case, indicates the relationship between the observed and the simulated shares of labor. If it is positive, implies that both variables move in the same direction. This implies that the greater (lower) is the data, the larger (smaller) are its simulations. On the contrary, a negative sign reflects that variables move in different directions. This implies that when one variable is high the other is low and vice-versa. When the coefficient is near to 1, data and simulations have a strong relationship. If the coefficient is near 0, the relationship between data and simulations is weak.

Table 4.4 show that the relationship between the simulated changes in the sectoral shares of labor and the observed ones is positive and high in cases like China, Egypt, Tanzania, India and the USA. It is remarkable the case of Senegal. For this country, correlation between data and simulations is negative in most sectors. Looking at the relative aggregate productivity, we can find that this statistic takes high positive values for most countries.

Considering all the statistics, it is noticeable that the model replicates better the data of countries that are in the first group. For Kenya and Tanzania the performance of the model is not so good, but it is not as bad as in the remaining countries of groups 2 and 3.

Chapter 5

Counterfactuals

This analysis is based on several experiments. It will reveal the relevance of the sectoral productivity growth on the aggregate productivity evolution of countries.
Firstly, we analyze what has been the importance of each sectoral productivity on the labor allocation and the relative aggregate productivity of countries. The experiments consist on replacing the sectoral productivity growth of countries by 0. As a result, we can obtain how would be the change in the labor allocation and the relative aggregate productivity when the productivity level of each sector remains remains constant. The next step is to compare these changes to the ones estimated for the countries with their corresponding sectoral productivity evolution. This comparison will allow us to see how was the influence of each sectoral productivity on the aggregate productivity evolution of a country. Furthermore, we will analyze how sectoral productivity has affected the income per capita evolution of each country.
Secondly, we analyze the relevance of specific productivity paths. For this purpose, we substitute the sectoral productivity evolution of a country by the sectoral productivity evolution of other country initially similar to it, in income per capita terms. Comparing the estimated changes for a country in the relative aggregate productivity and income per capita we can observe whether following other paths it could have been better or not .

5.1 Role of the sectoral productivity growth of countries in their relative aggregate productivity and income per capita

Table 5.1 presents the estimated changes in the relative aggregate productivity and income per capita of groups, given the observed sectoral productivity paths of countries. In addition, we can find how would have changed the sectoral labor allocation and the relative aggregate productivity of groups if, instead, some of the sectoral productivity of countries had remained constant[1]. It is necessary to remark that experiments are done assuming that any of the other factors different from the one analyzed do not change. This ensures that the results are only attributable to the factor that is modified. Therefore, it can be appreciated what is the role played by the productivity growth of each sector in the sectoral labor allocation and the aggregate productivity of groups.

Changes in the relative aggregate productivity and the income per capita of groups are expressed in percentage terms.

In this table, we can observe that there is a great difference on the change in the relative

[1]The corresponding information on the sectoral shares of labor can be found in Appendix C

Table 5.1: Importance of the sectoral productivity growth in the evolution of countries

	Group 1		Group 2		Group 3	
	Aggr. Productivity	GDPpc	Aggr. Productivity	GDPpc	Aggr. Productivity	GDPpc
Initial	546.67	1411.03	8.77	98.29	-12.90	61.33
Estimations when each sectoral productivity growth is set to 0						
AGR	190.52	597.59	-17.45	53.40	-48.98	-2.40
MIN	458.10	1198.77	8.52	97.85	-13.03	61.13
MAN	257.72	768.11	9.57	99.74	-6.79	72.55
PU	531.95	1376.57	8.67	98.11	-12.64	61.80
CON	477.10	1245.86	11.72	103.44	-11.13	64.65
WRT	421.74	1141.04	27.27	130.51	1.18	86.89
TRA	441.12	1168.31	9.25	99.11	-11.01	64.75
FIRE	476.47	1250.39	9.71	99.92	-11.52	63.79
OTH2	268.22	750.77	21.05	119.56	2.02	88.51

aggregate productivity of groups when considering that the agricultural productivity is constant and equal to its initial value. Since the change in the relative aggregate productivity is much lower when the agricultural productivity does not change, we may conclude that this sector has been one of the main determinants in the aggregate productivity growth of all groups. It is remarkable the importance of the productivity growth in agriculture for the second and the third group. If there had not been any growth in the agricultural productivity, the relative aggregate productivity of the second group would have experienced a negative evolution. In the third group, the negative growth of the relative aggregate productivity would have even been four times greater. Therefore, we may conclude that the growth in the agricultural productivity has contributed positively to the evolution of the relative aggregate productivity of groups.

It can be appreciated that the change in the relative aggregate productivity of group 1 would have been also lower without the increase in the manufacturing productivity or the personal and government productivity. Keeping any of these productivity at their initial level, would not have allowed the first group to reach more than half of the growth experienced in the relative aggregate productivity. The changes in income per capita of this group also reflects these sectoral productivity effects.

This table shows that the decrease in construction and in trade services experienced by group 2 have affected very negatively to the evolution of its relative aggregate productivity. If the productivity of any of these sectors had not decreased, the relative aggregate productivity and the income per capita of this group would have been much higher.

The initial estimations for group 2 differ from ones when the personal and government productivity is kept constant. However, we can see that the average annual growth of the producitivity in the initial estimations is also 0 and the remaining factors do not change. This is because in the first case, it is considered that both countries maintain constant their productivity, whereas in the second case, the countries considered experience very different growth rates, but, on average, they are 0. Looking at the estimations for its members, we can find that the evolution of the productivity in the personal and government sector has been very important in their relative aggregate productivity evolution. Indeed, if Ethiopia had not experienced any growth in the personal and government productivity, the relative aggregate productivity of this country would have experienced a higher decreased. In the case of Malawi, if the country had maintained constant this sectoral productivity, it could have achieved a much higher relative aggregate productivity. Therefore, we may conclude that the growth of the personal and government productivity is very important in determining the evolution of the relative aggregate productivity of countries in group 2.

The change in the relative aggregate productivity for group 3 would have been positive instead of negative, if its countries had not decreased their productivity in trade or personal and government services. This would have implied also an increase in its income per capita change.

5.2 The importance of specific sectoral productivity paths

Table 5.2 shows the estimated changes in the relative aggregate productivity and income per capita for some countries over the period analyzed, given their sectoral productivity evolution. In the last column are presented the estimated changes in the income per capita of countries. Below these figures we can find what would have been the changes in those variables, if the country had followed the sectoral productivity evolution of other countries or group of countries year by year[2].

In this table, we can see that if India had followed the sectoral productivity evolution of the other countries in group 1, this country would have experienced a variation in the relative aggregate productivity that would have almost doubled its own change. Indeed, by only following the agricultural productivity growth of group 1, India could have achieved a noticeable increase in the aggregate productivity. The productivity growth of mining experienced by group 1 alone could have also produced a greater change of the Indian aggregate productivity, though much lower than the produced by agriculture. Similarly, the income per capita of India would have experienced a much higher rate if the country had followed the sectoral productivity paths of group 1. At a lower extent, the income per capita of this country would have benefited from following the agricultural productivity growth of group 1. Much lower, but still positive, would have been if India had only followed the mining productivity path.

It can be appreciated that if Malawi had followed the sectoral productivity path of Egypt, its positive change in the relative aggregate productivity would have been much higher. Looking specifically to several sectors, we can find that by following Egypt's path in agriculture, Malawi could have achieved a greater increase in the relative aggregate productivity. This increase would have been lower, if instead, Malawi had followed the manufacturing productivity path of Egypt only. On the contrary, if Malawi had experienced the productivity path in services of Egypt, the country would have experienced a significantly higher growth rate in the relative aggregate productivity. The income per capita of Malawi would have had the same behavior than its relative aggregate productivity when experiencing some of the changes in sectoral productivity described previously.

When looking at how would have been Malawi if it had experienced Ethiopia's sectoral productivity growth, we find that it only would have been better, in terms of relative aggregate productivity and income per capita, when considering Ethiopia's services productivity path.

In the case of Ethiopia, it can be observed that the relative aggregate productivity change would have been positive if Ethiopia had experienced the sectoral productivity path of Malawi. The change in relative aggregate productivity would have been much greater if Ethiopia had only experienced Malawi's agricultural productivity. However, the change would have been worse than the experienced by Ethiopia if the country had followed Malawi's productivity growth in services. This is reflected on the estimations of the income per capita for Ethiopia. The greatest change is

[2] The corresponding information on the sectoral shares of labor can be found in Appendix C

Table 5.2: Consequences of following specific sectoral productivity paths

IND	Agg. Productivity	GDPpc
Initially	352.64	881.81
With the productivity path of BWA, CHN and EGY in...		
All sectors	661.72	1552.24
AGR	478.09	1153.93
MIN	358.65	894.84

MWI	Agg. Productivity	GDPpc
Initially	25.26	117.77
With the productivity path of EGY in...		
All sectors	703.70	1297.31
AGR	62.25	182.09
MAN	38.67	141.09
WRT-TRA-FIRE-OTH2	229.22	472.39

MWI	Agg. Productivity	GDPpc
Initially	25.26	117.77
With the productivity path of ETH in...		
All sectors	-33.09	16.32
AGR	-44.61	-3.70
TRA-FIRE-OTH2	148.86	332.67

ETH	Agg. Productivity	GDPpc
Initially	-7.71	78.82
With the productivity path of MWI in...		
All sectors	10.72	114.54
AGR	37.62	166.65
TRA-FIRE-OTH2	-9.17	75.99

TZA	Agg. Productivity	GDPpc
Initially	76.18	215.70
With the productivity path of KEN and SEN in...		
All sectors	-55.87	-20.93
PU	77.81	218.62
WRT-TRA-FIRE-OTH2	77.73	218.47

TZA	Agg. Productivity	GDPpc
Initially	76.18	215.70
With the productivity path of Group 1 in...		
All sectors	1455.52	2687.28
PU	150.92	349.62
MIN-MAN	114.36	284.10
WRT-TRA-FIRE-OTH2	491.83	960.49

KEN	Agg. Productivity	GDPpc
Initially	-35.14	23.66
With the productivity path of Group 1 in...		
All sectors	880.61	1769.72
PU	35.18	157.74
MIN-MAN	-19.40	53.69
WRT-TRA-FIRE-OTH2	56.09	197.61

SEN	Agg. Productivity	GDPpc
Initially	-79.75	-55.36
With the productivity path of Group 1 in...		
All sectors	-30.69	52.81
AGR	20.83	166.41

CHN	Agg. Productivity	GDPpc
Initially	1347.07	3164.61
With the productivity path of BWA in...		
All sectors	30.90	195.31
AGR	145.65	454.18
MIN	494.55	1241.32

estimated when Ethiopia follows only the agricultural productivity of Malawi. On the contrary, it is estimated that the change in income per capita would have been lower than the observed if Ethiopia had experienced the services productivity growth of Malawi.

Tanzania would have experienced a negative change in the relative aggregate productivity and the income per capita if it had followed the sectoral productivity evolution of the other countries of the third group. If Tanzania had only followed the services productivity growth, the country would have experienced increases significantly higher in these variables. On the contrary, following the same productivity growth rates in utilities as the other members of group 3 would have produced small positive changes in the relative aggregate productivity and the income per capita of the country.

Kenya and Tanzania show a higher increase in their relative aggregate productivity and income per capita when following the sectoral productivity path of the first group. The countries of the third group would have seen a considerable increase in their change of relative aggregate productivity if they had experienced the sectoral productivity evolution of group 1. When considering that they follow the productivity of group 1 in specific sectors, it can be observed that changes are not so great. In fact, the change in the relative aggregate productivity of Kenya is negative if the country only experiences the productivity growth of group 1 in industry. The greatest change is produced when the countries follow the same services productivity path as the first group.

The relative aggregate productivity and income per capita of Senegal behave in differently from the other members of the third group. For this country, experiencing the same sectoral productivity paths as the first group would have reduced the negative change in its relative aggregate productivity. However, it would imply a positive change in its income per capita. Senegal would have experienced high positive changes in its relative aggregate productivity and income per capita if instead, it had only experienced the agricultural productivity of the first group.

With respect to China, it is noticeable that changes in its relative aggregate productivity and income per capita would have been much lower following the sectoral productivity path of Botswana. This does not change much if we consider that China only follows the agricultural or mining productivity of Botswana.

Chapter 6

Conclusions

In this analysis, we have seen that there is not a single sector that explains the behaviour of the aggregate productivity and income per capita of groups 2 and 3. Both groups have decreased their productivity in most sectors. Similarly, there is not a single sector leading the evolution of group 1. This group has been able to achieve positive growth rates in the productivity of all sectors. However, the counterfactuals show that there has been specific sectors that have influenced a lot in the relative aggregate productivity and income per capita growth of countries.
Particularly, the experiments show that if the agricultural productivity had remained constant, the growth in relative aggregate productivity and income per capita of countries would have been much lower.
In addition, they reflect that the evolution of the personal and government services productivity has influenced a lot in the evolution of the aggregate productivity and income per capita of countries. If the productivity growth of this sector had been zero, countries in the first group would have experienced approximately half of the increase in the aggregate productivity and income per capita. On the contrary, if the second and third group had maintained constant their productivity in personal and government services, they would have improved noticeably the changes in aggregate productivity and income per capita.
The growth in the manufacturing productivity has affected very positively the relative aggregate productivity and income per capita of group 1. If the productivity growth in this sector had been 0, the growth of the relative aggregate productivity and income per capita of the first group would have been reduced in half. This sector was also very important for group 3. If this group had not decreased its manufacturing productivity, it would not have decreased so much its relative aggregate productivity and income per capita.
The trade services sector has been very important for the second and third groups. The negative evolution of the trade services productivity in these groups has impeded them to increase their relative aggregate productivity and income per capita as much as they could have done.

Looking at the individual experiments, we can appreciate that countries of group 3 would have increased noticeably their relative aggregate productivity and income per capita if they had followed the sectoral productivity growth of the first group. Indeed, by only following the agricultural or services productivity growth of the first group, they would have increased significantly the change in their relative aggregate productivity and income per capita. Similarly, India would have increased its relative aggregate productivity and income per capita much more if it had experienced a sectoral productivity path more alike to the one of the other countries in group 1. In the case of Malawi, this country could have improved noticeably the change in relative aggregate productivity and income per capita if it had experienced the sectoral productivity growth of Egypt.

There are some cases in which following the sectoral productivity path of a country with greater income per capita growth does not imply always an increase in the changes of relative aggregate productivity and income per capita. This might be because the initial importance of sectors differs among countries. This can be appreciated in the counterfactuals of Malawi with Ethiopia and Tanzania with the third group.

Malawi would have experienced a negative change in its income per capita and relative aggregate productivity if it had followed Ethiopia's productivity growth in all sectors or only in agriculture. However, if Malawi had only followed Ethiopia's productivity growth in some services (transport, business and personal and government services) it could have increased much more its relative aggregate productivity and income per capita.

Tanzania would have experienced a negative change in its income per capita and relative aggregate productivity if it had followed the productivity growth of the third group in all sectors. However, if Tanzania had only followed their productivity growth in some services (utilities and transport, business and personal and government services) it could have increased much more its relative aggregate productivity and income per capita.

The model used in this analysis assumes that countries allocate their labor force efficiently among all the sectors of the economy. The economies, in this model, are expected to devote a greater amount of their labor force to the sectors with greater productivity. However, mobility across sectors may not be so easy given that the skills of the labor demanded differ depending on the activity. Therefore, the allocation of labor force in countries may not be always efficient.

It is beyond the scope of this article to determine the factors that underlie the changes in the sectoral productivity of countries, the sectoral productivity of countries is taken as an exogenous variable. Therefore, we cannot conclude through this analysis that the causality between sectoral productivity and the aggregate productivity or income per capita of countries goes in one direction. It might be that an improvement in economic conditions allows countries to redistribute their resources and affect the sectoral productivity in future periods.

Another important aspect to keep in mind is that, had we included more countries in the sample, the greater the external validity of the study would have been. Analyzing more countries would allow us to ensure that the results we obtained could be applied to other countries that were not in our study but experienced an income per capita evolution similar to the ones described previously. This is because we could prove that what we observe is not a singular case observed for one country, but, on average, holds for a larger group. However, given the small number of countries studied in each group, we are aware that the results obtained from this analysis should only be used to explain the evolution of the countries included on it.

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Appendix A

The decision problem of households

The first step to solve the decision problem of households is to analyze how they distribute their consumption among all the non-agricultural goods and services in order to minimize its cost. Alternatively, it can be obtained solving the following maximization problem:

$$\max_{\{c_i\}_i \geq 0} c_i = \left(\gamma_1^{\frac{1}{\eta}} \cdot c_1^{\frac{\eta-1}{\eta}} + \ldots + \gamma_8^{\frac{1}{\eta}} \cdot c_8^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

$$\text{subject to } p_a \cdot c_a + \sum_{i=1}^{8} p_i \cdot c_i = 1$$

To solve this problem we will use the Lagrange multiplier

$$\mathcal{L} = \left(\sum_{i=1}^{8} \gamma_i^{\frac{1}{\eta}} \cdot c_i^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} + \lambda \cdot \left(1 - p_a \cdot c_a - \sum_{i=1}^{8} p_i \cdot c_i \right).$$

and differentiate it with respect to the different types of non-agricultural consumption

$$\frac{\partial \mathcal{L}}{\partial c_i} = \frac{\eta}{\eta - 1} \cdot \left(\sum_{i=1}^{8} \gamma_i^{\frac{1}{\eta}} \cdot c_i^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}-1} \cdot \gamma_i^{\frac{1}{\eta}} \cdot c_i^{\frac{\eta-1}{\eta}-1} \cdot \frac{\eta - 1}{\eta} - \lambda \cdot p_i = 0, \quad \forall i = 1, \ldots, 8$$

Those derivatives must be equal to each other and, through simple algebraic manipulation, we get an expression for $\frac{c_j}{c_i}$. Let $\mathcal{R} = \sum_{i=1}^{8} \gamma_i^{\frac{1}{\eta}} \cdot c_i^{\frac{\eta-1}{\eta}}$, and

$$\frac{\partial \mathcal{L}}{\partial c_i} = \frac{\partial \mathcal{L}}{\partial c_j}, \quad \forall i, j = 1, \ldots, 8 \text{ and } i \neq j.$$

Then,

$$\frac{\mathcal{R}^{\frac{\eta}{\eta-1}-1} \cdot \gamma_i^{\frac{1}{\eta}} \cdot c_i^{\frac{\eta-1}{\eta}-1}}{p_i} = \frac{\mathcal{R}^{\frac{\eta}{\eta-1}-1} \cdot \gamma_j^{\frac{1}{\eta}} \cdot c_j^{\frac{\eta-1}{\eta}-1}}{p_j} \Longrightarrow$$

$$\frac{\gamma_i^{\frac{1}{\eta}} \cdot c_i^{\frac{\eta-1}{\eta}-1}}{\gamma_j^{\frac{1}{\eta}} \cdot (c_j)^{\frac{\eta-1}{\eta}-1}} = \frac{p_i}{p_j} \Longrightarrow$$

$$\left(\frac{c_i}{c_j}\right)^{\frac{-1}{\eta}} = \frac{p_i}{p_j} \cdot \left(\frac{\gamma_i}{\gamma_j}\right)^{\frac{-1}{\eta}} \Longrightarrow$$

$$\frac{c_i}{c_j} = \left(\frac{p_j}{p_i}\right)^{\eta} \cdot \frac{\gamma_i}{\gamma_j},$$

which gives us

$$\frac{c_j}{c_i} = \left(\frac{p_i}{p_j}\right)^{\eta} \cdot \frac{\gamma_j}{\gamma_i}.$$

By considering the clearing condition for the labor market (equation 4.2), the production function (equation 4.1), the clearing condition for the goods market (equation 4.3), the equation for the relative non-agricultural consumption (equation 4.5) and the equation from the firm's decision problem (equation 4.4) we get the following expression for the non-agricultural shares of employment.

$$1 = L_i + L_a + \sum_{j=1,\, j\neq i}^{8} L_j \Longrightarrow$$

Dividing both sides by L_i

$$\frac{1}{L_i} = 1 + \frac{L_a}{L_i} + \sum_{j=1,\, j\neq i}^{8} \frac{L_j}{L_i} \Longrightarrow$$

Introducing the condition $Y_i = A_i \cdot L_i$,

$$\frac{1}{L_i} = 1 + \frac{L_a}{L_i} + \sum_{j=1,\, j\neq i}^{8} \frac{Y_j A_i}{Y_i A_j} \Longrightarrow$$

Because $c_a = Y_a$, $c_i = Y_i$,

$$\frac{1}{L_i} = 1 + \frac{L_a}{L_i} + \sum_{j=1,\, j\neq i}^{8} \frac{c_j A_i}{c_i A_j}.$$

Using the equation 4.5:

$$\frac{1}{L_i} = 1 + \frac{L_a}{L_i} + \sum_{j=1,\, j\neq i}^{8} \frac{A_i}{A_j}\left(\frac{p_i}{p_j}\right)^{\eta}\frac{\gamma_j}{\gamma_i} \Longrightarrow$$

$$\frac{1}{L_i} = 1 + \frac{L_a}{L_i} + \sum_{j=1,\, j\neq i}^{8} \frac{A_i}{A_j}\left(\frac{A_j}{A_i}\right)^{\eta}\frac{\gamma_j}{\gamma_i} \Longrightarrow$$

$$\frac{1}{L_i} = 1 + \frac{L_a}{L_i} + \sum_{j=1,\, j\neq i}^{8} \left(\frac{A_j}{A_i}\right)^{\eta-1}\frac{\gamma_j}{\gamma_i} \Longrightarrow$$

$$1 = L_i + L_a + \left(\sum_{j=1,\, j\neq i}^{8} \left(\frac{A_j}{A_i}\right)^{\eta-1}\frac{\gamma_j}{\gamma_i}\right) L_i \Longrightarrow$$

$$\left(\gamma_i A_i^{\eta-1} + \sum_{j=1,\, j\neq i}^{8} \gamma_j A_j^{\eta-1}\right) L_i = (1-L_a)\,\gamma_i A_i^{\eta-1} \Longrightarrow$$

$$L_i = \frac{(1-L_a)\,\gamma_i A_i^{\eta-1}}{\gamma_i A_i^{\eta-1} + \sum_{j=1,\, j\neq i}^{8} \gamma_j A_j^{\eta-1}}$$

which gives us an expression for

$$L_i = \frac{\gamma_i A_i^{\eta-1}(1-L_a)}{\sum_{i=1}^{8}\gamma_i A_i^{\eta-1}}.$$

Alternatively, this can be expressed as

$$1 = L_i + L_a + \sum_{j=1,\, j\neq i}^{8} \left(\frac{A_i}{A_j}\right)^{1-\eta}\frac{\gamma_j}{\gamma_i} L_i \Longrightarrow$$

$$\left(1 + \sum_{j=1,\, j\neq i}^{8} \left(\frac{A_i}{A_j}\right)^{1-\eta}\frac{\gamma_j}{\gamma_i}\right) L_i = 1 - L_a \Longrightarrow$$

$$L_i = \frac{1-L_a}{1 + \sum_{j=1,\, j\neq i}^{8} \left(\frac{A_i}{A_j}\right)^{1-\eta}\frac{\gamma_j}{\gamma_i}}.$$

Implementing the clearing condition for the goods and services market (equation 4.3) and the production function (equation 4.1), we can obtain an expression for the total consumption of non-agricultural goods and services in terms of the sectoral employment.

$$C = \left(\sum_{i=1}^{8} \gamma_i^{\frac{1}{\eta}} \cdot c_i^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \Longrightarrow$$

Implementing the clearing conditions for the goods and services markets

$$C = \left(\sum_{i=1}^{8} \gamma_i^{\frac{1}{\eta}} \cdot Y_i^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \Longrightarrow$$

Implementing the production function

$$C = \left(\sum_{i=1}^{8} \gamma_i^{\frac{1}{\eta}} \cdot (A_i \cdot L_i)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}.$$

Substituting the expressions for the sectoral employment obtained previously we can express the total consumption of non-agricultural goods in terms of the labor force in the agriculture sector, which will help us to determine the consumption of agricultural goods.

$$C = \left(\sum_{i=1}^{8} \gamma_i^{\frac{1}{\eta}} (A_i L_i)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \Longrightarrow$$

Using equation 4.6

$$C = \left(\sum_{i=1}^{8} \gamma_i^{\frac{1}{\eta}} \left(A_i \frac{\gamma_i A_i^{\eta-1} (1-L_a)}{\sum_{i=1}^{8} \gamma_i A_i^{\eta-1}} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \Longrightarrow$$

Simplifying

$$C = \left(\sum_{i=1}^{8} \gamma_i A_i^{\eta-1} \left(\frac{1-L_a}{\sum_{i=1}^{8} \gamma_i A_i^{\eta-1}} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \Longrightarrow$$

$$C = \left(\frac{\sum_{i=1}^{8} \gamma_i A_i^{\eta-1}}{\left(\sum_{i=1}^{8} \gamma_i A_i^{\eta-1} \right)^{\frac{\eta-1}{\eta}}} (1-L_a)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}.$$

Applying logarithms,

$$\ln C = \frac{\eta}{\eta-1} \left(\ln \left(\frac{\sum_{i=1}^{8} \gamma_i A_i^{\eta-1}}{\left(\sum_{i=1}^{8} \gamma_i A_i^{\eta-1} \right)^{\frac{\eta-1}{\eta}}} \right) + \frac{\eta-1}{\eta} \ln (1-L_a) \right) \Longrightarrow$$

$$\ln C = \left(\frac{\eta}{\eta-1} \ln \left(\frac{\sum_{i=1}^{7} \gamma_i A_i^{\eta-1}}{\left(\sum_{i=1}^{8} \gamma_i A_i^{\eta-1} \right)^{\frac{\eta-1}{\eta}}} \right) + \ln (1-L_a) \right).$$

Secondly, for a given combination of consumption of non-agricultural goods, we can solve for the optimal decision between the combination of all non-agricultural goods and the consumption of the agricultural good. For this purpose, we will substitute the expression for the combination of the non-agricultural consumption in the household's decision problem.

$$\max_{L_a \geq 0} U = \max_{L_a \geq 0} (1-a) \cdot \ln (c_a - \bar{a}) + a \cdot \ln \left[\gamma_1^{\frac{1}{\eta}} \cdot c_1^{\frac{\eta-1}{\eta}} + \ldots + \gamma_8^{\frac{1}{\eta}} \cdot c_8^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \Longrightarrow$$

Implementing the expression for the logarithm of the composite consumption

$$\max_{L_a \geq 0} U = \max_{L_a \geq 0} (1-a) \cdot \ln (c_a - \bar{a}) + a \cdot \frac{\eta}{\eta-1} \ln \left(\frac{\sum_{i=1}^{8} \gamma_i A_i^{\eta-1}}{\left(\sum_{i=1}^{8} \gamma_i A_i^{\eta-1} \right)^{\frac{\eta-1}{\eta}}} \right) + \ln (1 - L_a) \Longrightarrow$$

Implementing the clearing condition for the agricultural goods market (equation 4.3)

$$\max_{L_a \geq 0} U = \max_{L_a \geq 0} (1-a) \cdot \ln (Y_a - \bar{a}) + a \cdot \frac{\eta}{\eta-1} \ln \left(\frac{\sum_{i=1}^{8} \gamma_i A_i^{\eta-1}}{\left(\sum_{i=1}^{8} \gamma_i A_i^{\eta-1} \right)^{\frac{\eta-1}{\eta}}} \right) + \ln (1 - L_a) \Longrightarrow$$

Implementing the production function (equation 4.1)

$$\max_{L_a \geq 0} U = \max_{L_a \geq 0} (1-a) \cdot \ln (A_a L_a - \bar{a}) + a \cdot \frac{\eta}{\eta-1} \ln \left(\frac{\sum_{i=1}^{7} \gamma_i A_i^{\eta-1}}{\left(\sum_{i=1}^{8} \gamma_i A_i^{\eta-1} \right)^{\frac{\eta-1}{\eta}}} \right) + \ln (1 - L_a) .$$

Now, we can solve for the optimal decision by differentiating with respect to the labor in agriculture and obtain an expression for it.

$$\frac{\partial U}{\partial L_a} = \frac{(1-a) \cdot A_a}{A_a \cdot L_a - \bar{a}} - a \cdot \frac{1}{1 - L_a} = 0 \Longrightarrow$$

$$\frac{(1-a) \cdot A_a}{A_a \cdot L_a - \bar{a}} = a \cdot \frac{1}{1 - L_a} \Longrightarrow$$

$$(1 - L_a) \cdot (1-a) \cdot A_a = a \cdot (A_a \cdot L_a - \bar{a}) \Longrightarrow$$

$$(1-a) = a \cdot L_a + (1-a) \cdot L_a - \frac{a \cdot \bar{a}}{A_a} \Longrightarrow$$

$$L_a = (1-a) + \frac{a \cdot \bar{a}}{A_a}.$$

Appendix B

Market clearing conditions

The expressions we have for the sectoral employment are:

$$L_i = \frac{\gamma_i \cdot A_i^{\eta-1} \cdot (1 - L_a)}{\sum_{i=1,\ldots,8} \gamma_i \cdot A_i^{\eta-1}}, \quad \text{for } i = 1, \ldots, 8.$$

From the labor market clearing condition we know that

$$1 = L_a + \sum_{i=1,\ldots,8} L_i.$$

Introducing our sectoral employment expressions

$$\begin{gathered}
1 = L_a + \frac{\sum_{i=1,\ldots,8} \gamma_i \cdot A_i^{\eta-1} \cdot (1 - L_a)}{\sum_{i=1,\ldots,8} \gamma_i \cdot A_i^{\eta-1}} \Longrightarrow \\
1 = L_a + \frac{\sum_{i=1,\ldots,8} \gamma_i \cdot A_i^{\eta-1}}{\sum_{i=1,\ldots,8} \gamma_i \cdot A_i^{\eta-1}} (1 - L_a) \Longrightarrow \\
1 = L_a + 1 - L_a \Longrightarrow \\
1 = 1.
\end{gathered}$$

The alternative expressions we have for the sectoral employment are:

$$L_i = \frac{1 - L_a}{1 + \sum\limits_{j=1,\, j\neq i}^{8} \left(\frac{A_i}{A_j}\right)^{1-\eta} \frac{\gamma_j}{\gamma_i}}, \quad \text{for } i = 1, \ldots, 8.$$

From the labor market clearing condition we know that

$$1 = L_a + \sum_{i=1,\, i\neq j}^{8} L_i + L_j.$$

So, introducing our alternative sectoral employment expressions

$$1 = L_a + \sum_{i=1,\, i\neq j}^{8} \frac{1 - L_a + \frac{\bar{s}}{A_8}}{1 + \sum\limits_{j=1,\, j\neq i}^{8} \left(\frac{A_i}{A_j}\right)^{1-\eta} \frac{\gamma_j}{\gamma_i}} + \left(1 - L_a - \sum_{i=1,\, i\neq j}^{8} \frac{1 - L_a}{1 + \sum\limits_{j=1,\, j\neq i}^{8} \left(\frac{A_i}{A_j}\right)^{1-\eta} \frac{\gamma_j}{\gamma_i}}\right) \Longrightarrow$$

$$1 = L_a + 1 - L_a \Longrightarrow$$

$$1 = 1.$$

Given that the clearing condition of the labor market is fulfilled in both cases, we can rely on our expressions for the sectoral employment.

Appendix C

Counterfactuals: changes in the sectoral shares of employment

The following tables show the changes in the shares of employment allocated to each sector by groups. In addition, it presents how would be the changes if the productivity growth of each sector had been 0.

Group 1	L_{AGR}	L_{MIN}	L_{MAN}	L_{PU}	L_{CON}	L_{WRT}	L_{TRA}	L_{FIRE}	L_{OTH2}
Initial	-48.67	41.17	130.38	127.63	325.59	264.32	78.40	133.18	111.72
AGR	0.00	-31.64	29.96	10.98	86.85	84.52	-19.98	24.97	-12.73
MIN	-48.67	491.97	102.96	102.05	278.62	216.10	57.81	112.38	82.21
MAN	-48.67	-11.08	273.43	40.90	148.16	143.52	3.73	63.36	13.20
PU	-48.67	38.56	126.09	574.48	316.60	256.93	74.79	129.11	107.06
CON	-48.67	31.71	109.22	103.50	515.27	227.50	62.29	116.92	88.77
WRT	-48.67	18.97	111.46	93.44	252.76	468.23	48.29	105.06	73.43
TRA	-48.67	22.54	104.23	96.65	263.72	216.63	481.33	105.68	78.26
FIRE	-48.67	35.11	121.23	115.55	303.14	245.47	68.39	543.03	97.32
OTH2	-48.67	-11.46	34.99	39.44	165.86	116.05	10.61	42.95	322.08

Group 2	L_{AGR}	L_{MIN}	L_{MAN}	L_{PU}	L_{CON}	L_{WRT}	L_{TRA}	L_{FIRE}	L_{OTH2}
Initial	-16.08	-15.30	12.60	-24.11	88.54	195.35	8.00	28.51	92.19
AGR	0.00	84.14	0.28	-39.14	82.67	50.68	-46.28	-26.76	-23.88
MIN	-16.08	-2.11	12.49	-24.21	88.41	194.72	7.76	28.26	91.70
MAN	-16.08	-8.49	0.47	-21.98	95.04	200.35	9.78	30.98	94.60
PU	-16.08	-15.37	12.42	-2.12	88.26	194.84	7.81	28.29	91.84
CON	-16.08	-10.19	19.28	-19.61	3.85	212.82	14.38	36.12	103.53
WRT	-16.08	4.16	45.58	-1.55	143.05	27.59	41.73	68.19	153.16
TRA	-16.08	-16.02	13.00	-23.78	89.08	197.48	-1.45	29.37	93.77
FIRE	-16.08	-15.28	12.89	-23.90	89.01	196.33	8.36	-1.67	92.87
OTH2	-16.08	-18.24	26.47	-13.93	109.96	245.56	26.53	49.42	12.21

Group 3	L_{AGR}	L_{MIN}	L_{MAN}	L_{PU}	L_{CON}	L_{WRT}	L_{TRA}	L_{FIRE}	L_{OTH2}
Initial	-0.18	6.56	63.40	63.68	78.97	88.48	47.25	44.16	71.13
AGR	13.06	-41.94	-31.88	-45.08	-4.58	-26.26	-47.70	-53.68	-34.47
MIN	-0.18	0.01	63.26	63.19	79.33	88.19	46.89	43.69	70.83
MAN	-0.18	15.22	6.40	72.99	93.27	100.43	55.97	52.13	81.81
PU	-0.18	6.79	63.90	0.53	79.37	89.08	47.76	44.69	71.68
CON	-0.18	10.66	67.43	66.26	2.25	92.60	49.89	46.22	74.71
WRT	-0.18	24.06	91.73	93.00	108.51	17.85	73.43	70.14	101.21
TRA	-0.18	8.49	67.08	67.81	82.28	92.88	2.59	47.88	75.17
FIRE	-0.18	6.72	64.13	64.71	79.29	89.43	48.12	0.74	72.02
OTH2	-0.18	26.47	92.38	91.71	112.27	121.55	72.69	68.70	17.68

The following table show the change in the share of labor allocated to each sector by some countries. In addition, it presents which would have been the changes if the countries had followed the sectoral productivity path of others.

	L_{AGR}	L_{MIN}	L_{MAN}	L_{PU}	L_{CON}	L_{WRT}	L_{TRA}	L_{FIRE}	L_{OTH2}
MWI									
Initial	-39.12	-40.89	79.51	25.69	190.28	449.34	101.84	133.67	272.77
With the productivity path of EGY in...									
All sectors	-63.29	261.01	355.56	713.43	357.99	495.97	292.72	531.83	253.66
AGR	-63.29	-15.68	156.07	79.29	314.10	683.65	187.93	233.33	431.77
MAN	-39.12	-35.49	-16.44	37.16	216.78	499.48	120.26	154.99	306.79
WRT-TRA-FIRE-OTH2	-39.12	40.68	327.21	199.12	590.85	138.39	57.09	152.73	41.46
With the productivity path of ETH in...									
All sectors	6.96	69.74	-29.69	-59.85	33.59	-36.34	-78.20	-64.05	-82.15
AGR	6.96	-88.95	-66.45	-76.51	-45.75	2.66	-62.28	-56.33	-30.34
WRT-TRA-FIRE-OTH2	-39.12	11.29	237.98	136.64	446.55	369.89	60.90	165.39	31.78
ETH									
Initial	6.96	10.29	-54.31	-73.91	-13.20	-58.64	-85.84	-76.64	-88.40
With the productivity path of MWI in...									
All sectors	-38.51	-24.53	130.16	52.48	260.19	591.95	148.70	145.86	349.37
AGR	-38.51	1575.44	594.03	296.36	1218.67	528.37	115.17	254.90	76.22
TRA-FIRE-OTH2	6.96	-28.59	-70.42	-83.11	-43.80	-73.22	-79.67	-79.90	-63.27
TZA									
Initial	-34.42	58.93	253.94	324.71	177.96	333.77	266.89	284.93	301.32
With the productivity path of KEN and SEN in...									
All sectors	6.50	-51.57	-50.30	-90.71	-48.80	-41.93	-71.80	-76.65	-61.71
PU	-34.42	60.11	256.56	-12.13	180.02	336.98	269.60	287.77	304.29
WRT-TRA-FIRE-OTH2	-34.42	55.51	246.32	315.57	171.98	433.60	159.15	114.59	251.83
With the productivity path of group 1 in...									
All sectors	-57.82	182.84	471.85	421.81	723.65	634.55	315.03	487.26	356.67
AGR	-57.82	139.03	432.32	538.77	318.05	552.39	451.80	478.93	503.59
MIN-MAN	-34.42	-51.59	-2.13	378.40	213.10	388.60	313.26	333.58	352.05
WRT-TRA-FIRE-OTH2	-34.42	322.26	840.38	1028.42	638.52	196.55	67.55	137.08	84.36

	L_{AGR}	L_{MIN}	L_{MAN}	L_{PU}	L_{CON}	L_{WRT}	L_{TRA}	L_{FIRE}	L_{OTH2}
KEN									
Initial	-5.28	60.74	36.27	-33.69	158.95	31.68	-25.12	-52.43	12.06
With the productivity path of group 1 in...									
All sectors	-57.75	67.67	239.00	209.34	388.28	335.45	146.04	248.14	170.72
AGR	-57.75	335.77	269.44	79.77	602.02	256.97	102.99	28.95	203.80
MIN-MAN	-5.28	-86.21	-72.11	-19.84	213.04	59.18	-9.49	-42.50	35.47
WRT-TRA-FIRE-OTH2	-5.28	269.59	213.33	52.47	495.41	-31.87	-61.50	-45.53	-57.64
SEN									
Initial	39.17	-100.00	-100.00	-100.00	-100.00	-100.00	-100.00	-100.00	-100.00
With the productivity path of group 1 in...									
All sectors	39.17	-100.00	-100.00	-100.00	-100.00	-100.00	-100.00	-100.00	-100.00
AGR	-57.66	148.67	159.19	-77.18	57.29	264.90	60.32	64.49	85.55
MIN-MAN	39.17	-100.00	-100.00	-100.00	-100.00	-100.00	-100.00	-100.00	-100.00
WRT-TRA-FIRE-OTH2	39.17	-100.00	-100.00	-100.00	-100.00	-100.00	-100.00	-100.00	-100.00
IND									
Initial	-39.14	106.15	81.26	18.57	556.74	150.05	79.13	204.44	42.54
With the productivity path of BWA, CHN and EGY in...									
All sectors	-61.92	7.47	177.83	186.45	200.52	255.62	82.49	149.12	121.74
AGR	-61.92	167.20	134.94	53.69	751.22	224.10	132.18	294.60	84.75
MIN	-39.14	-32.43	83.36	19.95	564.33	152.94	81.20	207.96	44.19
CHN									
Initial	-76.23	78.22	125.00	281.08	627.36	465.61	228.70	107.40	402.21
With the productivity path of BWA in...									
All sectors	-16.71	-44.48	259.89	29.78	60.64	338.14	-29.91	103.27	-19.90
AGR	-16.71	0.75	27.19	115.42	311.17	219.74	85.82	17.25	183.90
MIN	-76.38	417.53	309.45	593.46	1223.60	929.26	498.16	277.42	813.90